Evaluation of a Graph Coloring based Satellite Resource Allocation Scheme

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***Abstract*—This paper evaluates resource allocation (RA) schemes for low Earth orbit (LEO) satellite systems. In traditional multibeam satellite systems with frequency reuse, severe spectral inefficiency can arise when traffic distributions become highly dynamic and heterogeneous, a situation often encountered in LEO networks. In such scenarios, multibeam systems with frequency overlap can be an effective approach, at the cost of increased computational complexity for determining the optimal bandwidth and power allocation. To address this challenge, this paper proposes an RA method based on graph coloring theory, which offers an efficient means of reducing the computational complexity involved in finding optimal allocations. Simulation results demonstrate that the proposed scheme not only lowers computational complexity but also improves performance compared with conventional approaches.**

***Index Terms*—LEO satellite, resource allocation, heterogeneous traffic, graph coloring theory, frequency reuse**

I. Introduction

The satellite communication system has been gaining significant attention in recent years because of its wide-area coverage and its ability to provide continuous connectivity. In particular, low Earth orbit (LEO) satellites, with their advantages of low launch cost and low latency, are expected to play an important role in the non-terrestrial network (NTN) of sixth-generation (6G) wireless communication scenarios [1], [2]. Scarcity of satellite power and bandwidth resources makes resource allocation (RA) strategies extremely important. Dynamic RA schemes for multibeam satellite systems have been proposed by utilizing a power–bandwidth linear system for optimal bandwidth and power allocation [3]–[5]. However, these studies assumed that the traffic demand (TD) is distributed homogeneously across all satellite beams in RA coverage, which is an impractical assumption under the operational conditions of a LEO satellite constellation which covers the entire Earth.

Numerous studies have been proposed to address RA problem under heterogeneously distributed TD [6]–[8]. These studies proposed RA schemes that allow overlap between frequency bands within the permissible level of interference. However, finding the optimal allocation via reinforcement learning or greedy-based methods poses substantial computational burden.

In recent studies, graph coloring theory which traditionally was adopted to solve spectrum management problem in cognitive radio networks, have been applied to the satellite network spectrum allocation [9], [10]. Building upon previous investigations, this paper proposes a dynamic RA scheme for multibeam LEO satellite systems and compares the performance of different RA methods under the heterogeneous TD distribution environment. In this study, the bandwidth and power allocation problem is tackled with graph coloring techniques, by modeling possible position of beams as undirected graph.

Following this introduction, this paper is organized as follows: Section II reviews the related work on RA schemes for multibeam LEO satellite systems. Section III proposes an RA scheme based on the graph coloring method and frequency reuse technique. Section IV provides simulation results, and conclusions are drawn in Section IV.

II. Related Works

RA methods for interference-limited satellite systems have been proposed assuming that the multibeam satellite systems reuse the total frequency bandwidth of W [Hz], with a frequency reuse factor of F [4], [5]. these studies focused on minimizing power consumption while maximizing system capacity with the following optimization problem:

$$\arg\min_{\mathbf{w}} P_{\text{total}} \quad \text{s.t.} \quad \begin{aligned} &\sum_{i=1}^{F} w_i \leq W, \quad w_i \geq 0, \\ &(P_t)_i^j \geq 0, \quad \forall i, j, \\ &P_{\text{total}} \leq P_{\max}, \end{aligned} \tag{1}$$

where $\mathbf{w} = [w_1 \ w_2 \ \cdots \ w_i \cdots \ w_F]$ is a bandwidth vector and P_{total} [Watt] denotes the total transmit power. In addition, $(P_t)_i^j$ represents the transmit power of the jth beam using w_i and $P_{\max}$ denotes the maximum transmit power.

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To obtain the optimal $\mathbf{w}$ by finding the solution of (1), the following linear system which defines the relationship between frequency bandwidth and power was established [4]:

$$\mathbf{\Delta}_i \left[\gamma_i^1 \ \gamma_i^2 \ \cdots \ \gamma_i^j \ \cdots \ \gamma_i^{M_i}\right]^T = \left[1 \ 1 \ \cdots \ 1 \ \cdots \ 1\right]^T, \quad (2)$$

where γ_i^j denotes the bit energy to noise spectral density ratio, $(E_b)_i^j/N_0$ required at beam j using w_i, and M_i denotes the number of beams using w_i, and

$$\mathbf{\Delta}_i = \begin{bmatrix} 1/\rho_i^1 & -g_i^{2,1}\eta_i^2 & \cdots & -g_i^{M_i,1}\eta_i^{M_i} \\ -g_i^{1,2}\eta_i^1 & 1/\rho_i^2 & \cdots & -g_i^{M_i,2}\eta_i^{M_i} \\ \vdots & \vdots & \ddots & \vdots \\ -g_i^{1,M_i}\eta_i^1 & -g_i^{2,M_i}\eta_i^2 & \cdots & 1/\rho_i^{M_i} \end{bmatrix}, \quad (3)$$

where $\rho_i^j = (E_b)_i^j/((I_0)_i^j + N_0)$ denotes the required bit energy to interference plus noise spectral density ratio for beam j using w_i while $(I_0)_i^j$ denotes the interference spectral density at beam j using w_i. In addition, $g_i^{k,j}$ is the relative link gain, which can be represented as follows:

$$g_i^{k,j} = \frac{(G_t)_i^{k,j} h_i^{k,j} (G_r)_i^{k,j}}{(G_t)_i^{k,k} h_i^{k,k} (G_r)_i^{k,k}}, \quad (4)$$

where $(G_t)_i^{k,j}$ and $(G_r)_i^{k,j}$ represent the transmit and receive antenna gains in the direction from beam k to beam j using w_i, respectively, while $(G_t)_i^{k,k}$ and $(G_r)_i^{k,k}$ correspond to the antenna gains for the desired signal path of beam k using w_i, respectively. In addition, $h_i^{k,j}$ denotes the channel gain across the path from beam k to beam j using w_i, including free space loss, rain attenuation, and other fading and losses. Finally, η_i^j [bps/Hz] is defined by Shannon's capacity equation as follows [4]:

$$\eta_i^j = \frac{(R_b)_i^j}{w_i} = \log_2\left(\frac{(P_r)_i^j}{I_i^j + N_i} + 1\right), \quad (5)$$

where $(R_b)_i^j$ [bps] denotes a TD from beam j using w_i and $(P_r)_i^j$ is received power for beam j using w_i, respectively. In addition, $I_i^j = (I_0)_i^j w_i$ and $N_i = N_0 w_i$ are the interference and noise power at beam j using w_i, respectively.

By representing ρ_i^j in terms of power ratio, we can find a dependent relationship between ρ_i^j and η_i^j as follows:

$$\begin{aligned} \rho_i^j &= \frac{(E_b)_i^j}{(I_0)_i^j + N_0} = \frac{(P_r)_i^j/(R_b)_i^j}{I_i^j/w_i + N_i/w_i} \\ &= \frac{(P_r)_i^j}{I_i^j + N_i} \cdot \frac{w_i}{(R_b)_i^j} = \frac{(P_r)_i^j}{I_i^j + N_i} \cdot \frac{1}{\eta_i^j}. \end{aligned} \quad (6)$$

From (5), $\frac{(P_r)_i^j}{I_i^j + N_i} = 2^{\eta_i^j} - 1$ and (6) can be rewritten as follows:

$$\rho_i^j = \frac{2^{\eta_i^j} - 1}{\eta_i^j}, \quad (7)$$

and $(P_t)_i^j$ can be derived from the solution of (2). Since a closed-form solution of (2) is not available, an iterative search method is required to find the solution. To reduce the complexity of iterative search, previous work has proposed RA strategies utilizing simple machine learning and deep learning techniques [3]–[5]. However, the RA scheme that relies on a fixed frequency reuse pattern may not fully utilize the available resources when the TD distribution is highly heterogeneous.

III. Proposed Resource Allocation Scheme Using Graph Coloring Method

In this paper, we consider highly heterogeneous TD environment, and cells with TD are referred to as active cells. Considering only active cells with effective interference, the the maximum dimensionality of the problem in (2) can be reduced from a third-order formulation of size $M_m \times M_m \times F$ to a second-order of size $B \times B$, where M_m is the maximum number of cells using the same frequency band and B is the number of active cells. Since $B < M_i, \exists\ i,\ 1 \leq i \leq F$, there will be huge reduction in searching range of the optimal bandwidth allocation. Furthermore, bandwidths for each active cell can be allocated in a manner that allows overlap between frequency bands within the permissible level of interference, that is, the bandwidth allocation vector can be composed as $\mathbf{w} = [w_1 \ w_2 \ \cdots \ w_B]$ of size $1 \times B$. Accordingly, we have a matrix $\mathbf{\Delta}$ with a size of $B \times B$ for $\mathbf{w}$, resulting in optimal transmit power allocation for all active cells.

As in the previous studies mentioned in Section II, an iterative search-based algorithm is needed in order to find the optimum $\mathbf{w}$ which can minimize P_{total}. However, searching all possible candidates requires high computational complexity, and it is also easy to miss the optimum $\mathbf{w}$ that achieves the global minimum of P_{total}. In order to address these problems, we adopt the graph coloring method which is one of the traditional methods solving channel allocation problem in cognitive radio networks [11].

By applying the graph coloring method, the network is modeled as an undirected graph $G = (V, E)$ and vertices (V) represent users, edges (E) represent interference [12]. In a multibeam satellite system, the position of the beam (or cell) centers can be seen as V and adjacent cells can be connected by E, so that adjacent V do not use the same color, which corresponds to adjacent cells cannot utilize the same bandwidth [13]. Fig. 1 illustrates an example of bandwidth allocation with graph coloring method where active cells modeled as an undirected graph.

In the system shown in Fig. 1, the center points of active cells are considered as V and each cell is indexed from 1 to B. Then, E can be designated according to the location of V. For instance, vertex 5, with no adjacent active cells, has no incident edges to any other vertices. We denote the number of colors used in the coloring by N, which corresponds to the frequency reuse factor, i.e., $N = 3$ in Fig. 1. That is, N sub-bands are reused among the beams, where the n-th sub-band is denoted by w_n, $n = 1, 2, ..., N$. Furthermore, let M_n represent the number of beams using the same sub-band w_n.

With the above, the optimization problem in (1) can be modified to minimize transmit power consumption and to

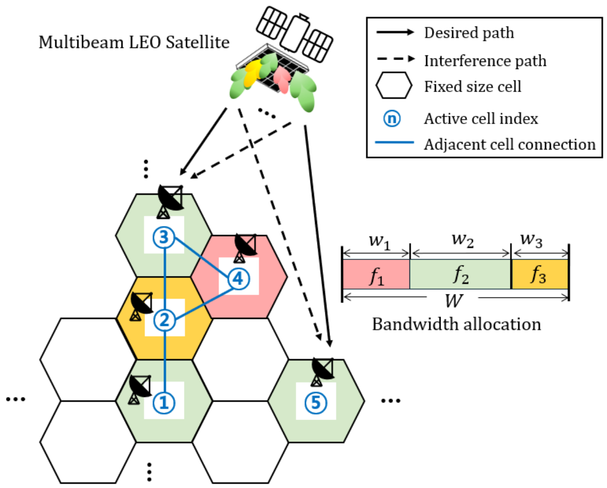

Fig. 1. An example of graph coloring based resource allocation

maximize system capacity as follows:

$$\underset{\mathbf{w}}{\arg\min}\ P_{\text{total}} \quad \text{s.t.} \quad \begin{array}{l} \sum_{n=1}^{N} w_n = W, \quad w_n > 0 \\ (P_t)_n^l \geq 0, \quad (R_b)_n^l > 0, \quad \forall n, l, \\ P_{\text{total}} \leq P_{\max}, \end{array} \tag{8}$$

where $(P_t)_n^l$ and $(R_b)_n^l$ denote transmit power and aggregated TD at beam l using w_n when $l = 1, 2, ..., M_n$, respectively. By applying the coloring method, the total system bandwidth W is divided into N sub-bands, where N is the number of colors used, i.e., the chromatic number. Therefore, the linear system relating the power and bandwidth can be expressed as follows:

$$\boldsymbol{\Delta}_n \left[\gamma_n^1 \ \gamma_n^2 \ \cdots \ \gamma_n^l \cdots \gamma_n^{M_n}\right]^T = \left[1 \ 1 \cdots 1 \cdots \ 1\right]^T, \tag{9}$$

where

$$\boldsymbol{\Delta}_n = \begin{bmatrix} 1/\rho_n^1 & -g_n^{2,1}\eta_n^2 & \cdots & -g_n^{M_n,1}\eta_n^{M_n} \\ -g_n^{1,2}\eta_n^1 & 1/\rho_n^2 & \cdots & -g_n^{M_n,2}\eta_n^{M_n} \\ \vdots & \vdots & \ddots & \vdots \\ -g_n^{1,M_n}\eta_n^1 & -g_n^{2,M_n}\eta_n^2 & \cdots & 1/\rho_n^{M_n} \end{bmatrix}. \tag{10}$$

IV. Simulation Results

In this section, we investigate the performance of the proposed RA scheme using the graph coloring method and compare it with the conventional methods which utilized linear system models, including a dynamic RA method in [4] and our previous work in [8]. We refer to the conventional methods in [4] and [8] as Conventional 1 and Conventional 2, respectively. It is assumed that RA is performed for 9 LEO satellite beams and the number of active cells, $B = 5$. The satellite altitude and the cell radius are assumed to be 1200 km and 50 km, respectively. The center frequency and W are set to 18.5 GHz and 200 MHz, respectively. $P_{\max}$ and N_0 are set to 17.5 dBW and -203.3 dBW/Hz, respectively. The antenna gains of the satellite and Earth station are set to be 37 dBi and 43.5 dBi, respectively.

In addition, for Conventional 1, $F = 3$ and $M_i = 3$. The aggregated TD of each cell in both the conventional schemes and proposed scheme is assumed to follow Gaussian distribution, $\mathcal{G}(\mu, \sigma^2)$ (Mbps) where $\mu = [300, 500]$ (Mbps), $\sigma = 0.2\mu$ and $\mu \geq 0$. In the proposed scheme, we emulate scenarios when N can be either 2 or 3 depending on the location of active cells.

Fig. 2 compares the successful allocation rate. It shows that the proposed method achieves approximately 14 percentage points higher success rate than Conventional 1, and shows approximating performance to Conventional 2, when $E[R_b] = 400$ (Mbps). Fig. 3 shows TD versus supplied traffic through the utilized RA schemes, and it also shows that the proposed method provides enhanced traffic supply compared to the conventional methods when $E[R_b] = 400$ (Mbps). Fig. 4 compares the P_{total} (dBW) under the successful allocation cases. At $E[R_b] = 400$ Mbps, the proposed method and Conventional 2 respectively reduce P_{total} by approximately 2.5 dBW and 3.2 dBW, compared to Conventional 1. This power reduction is achieved through the comparatively wider bandwidth allocation of the proposed method.

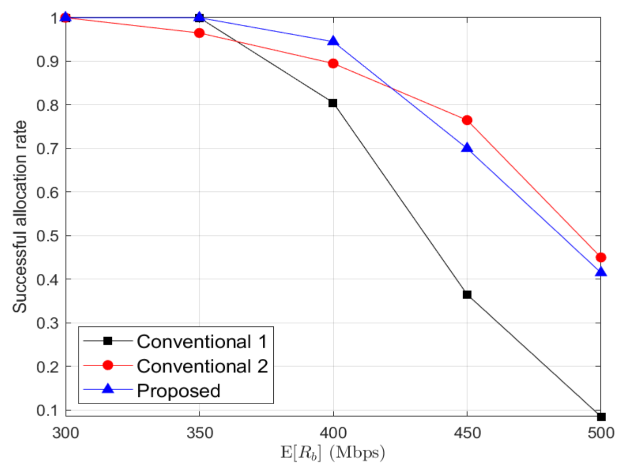

Fig. 2. Comparison of successful resource allocation rate.

V. Conclusion

In this paper, we proposed a RA scheme based on the graph coloring method for multibeam LEO satellite systems under heterogeneous traffic patterns. The simulation results compared different RA schemes and showed that the proposed method can enhance the system efficiency with low computational complexity. The current simulation assumed restricted setting of TD patterns and cells. In the future, our study will be focused on developing an algorithm that is more suitable for the application of graph coloring theory and considering practical environments of cells and TDs.

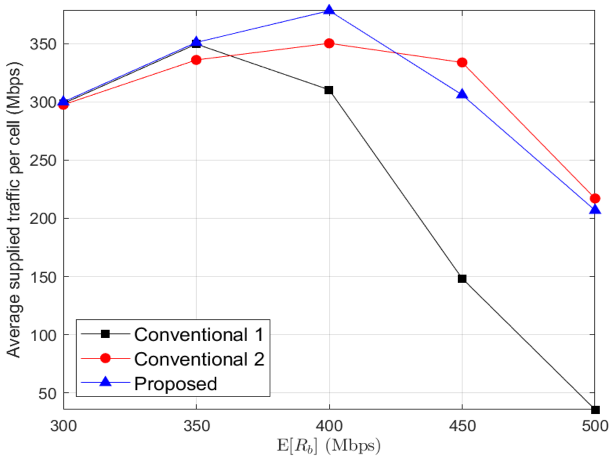

Fig. 3. Comparison of supplied traffic per cell.

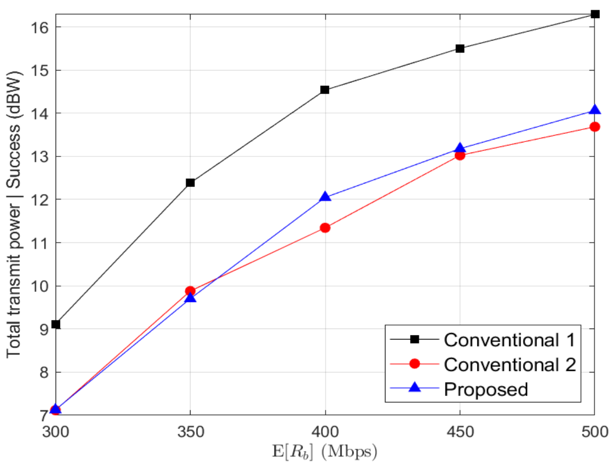

Fig. 4. Comparison of average total transmit power under the successful resource allocation.

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