UAV-BS Assisted User Localization Considering Bearing Observability in mmWave Systems

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***Abstract*—Recently, millimeter-wave (mmWave) technology has enabled accurate user localization from a single ground reference station. However, in unmanned aerial vehicle base station (UAV-BS) assisted mmWave systems, the user localization quality is significantly affected by the varing bearing observability as a result of the UAV movement. In this paper, to address the observability issue, we first propose a user localization method based on an unscented kalman filter (UKF). The states of the geometric relationship between the user and UAV-BS, comprising the relative location and velocity, are estimated, and these construct the bearing observability metrics for user localization. Thereafter, the optimal message interval that maximizes the observability criterion is investigated. Simulation results demonstrate that the proposed method provides higher accuracy for user position and orientation.**

***Index Terms*—Bearing observability, millimeter-wave, optimization, unmanned aerial vehicle system, localization.**

I. Introduction

The millimeter-wave (mmWave) multiple-input-multiple-output (MIMO) technology, where both users and a base station (BS) are equipped with arrays of a large number of antennas, becomes a strong candidate to support explosively increased location-aided applications [1]. Owing to the high resolution of both the angular and temporal domains, time of arrival (TOA), angle of arrival (AOA) and angle of departure (AOD) estimations enable promising two-dimensional (2D) and three-dimensional (3D) localization accuracies from a single ground BS. [2], [3].

The main difference in localization challenges between a unmanned aerial vehicle base station (UAV-BS) system and a regular cellular BS system is whether the BS can move [4]. Owing to its agility and mobility, UAV-BS can provide several different observation points that can be used as an advantage for range-measurement-based localization. [5] studied a localization solution in conventional UAV cellular systems considering a mixture of line-of-sight (LoS) and non-LoS (NLoS) based signal power models using the least square (LS) algorithm, which uses different range measurements from multiple observation points.

In mmWave localization, however, bearing measurements such as AOA and AOD are critical, and rapidly changing observation points can pose significant challenges [6]. This issue becomes more pronounced in UAV-assisted systems, where bearing observability varies more rapidly due to continuous changes in the relative position between the user and the UAV-BS. Although system observability is a crucial factor affecting the performance of state estimation algorithms, to the best of our knowledge, limited research has addressed the observability problem in UAV-BS mmWave localization.

In this paper, we propose a states estimation method that reflects the geometric relationship between the user and UAV-BS by using an unscented kalman filter (UKF) approach. The estimated relative movements of the system are employed to construct the bearing observability metrics. Then the optimal message interval required for maximizing bearing observability is derived, and this information is sent to the UAV-BS as feedback. Simulation results demonstrate that the proposed method can provide higher localization accuracy and sufficient performance with respect to relatively high AOA and AOD measurements noises.

II. System Model

A. System Geometry

Fig. 1 shows the geometry of the proposed system. We consider a UAV-BS with an array of N_T antennas and a user equipped with N_R antennas operating at a carrier frequency f_c, whose corresponding wavelength is λ_c and bandwidth B. We assume that the UAV-BS maintains a constant altitude, i.e., it moves at a fixed height. For simplicity, only 2D motion is considered, which can be readily extended to a 3D model. Let ($\mathbf{p}_u$, φ_u, v_u) and ($\mathbf{p}_U$, φ_U, v_U) denote the location, heading and velocity of the user and UAV-BS, respectively, in the Cartesian coordinate system. The positions of the user and UAV are expressed in the global frame of reference as $\mathbf{p}_u = [x_u, y_u]^T \in \mathbb{R}^2$ and $\mathbf{p}_U = [x_U, y_U]^T \in \mathbb{R}^2$, respectively. The headings of the UE and UAV are defined to be positive in the clockwise direction with respect to the y-axis and range from $-\pi < \varphi \leq \pi$. Both the UE and UAV-BS are assumed to move with constant velocity and constant turn rate.

The user estimates $\mathbf{p}_u$ and φ_u by measuring AOA, AOD and TOA from the UAV-BS. We denote the AOA and AOD as φ_R and φ_T, respectively. The movement of UAV-BS makes its states change continuously; hence, the UAV-BS assists user localization by periodically sending messages including $\mathbf{p}_U$, φ_U, and v_U. The LoS probability of a UAV-BS to the user link is significantly higher than that of a ground BS to the user link due to the UAV elevation height. Therefore, we only consider the LoS link in this paper.

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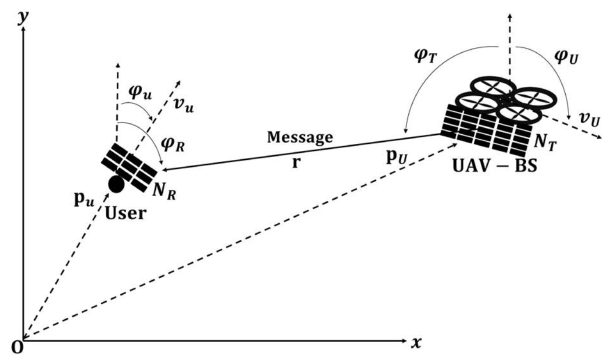

Fig. 1. Two dimensional illustration of the proposed system geometry.

B. Transmission Model

We consider the transmission of orthogonal frequency division multiplexing (OFDM) signals as in [7]. The UAV-BS sequentially transmits G signals, where the g-th transmission comprises L symbols $\mathbf{x}^{(g)}[n] = [x_1[n], ..., x_L[n]]^{\mathbf{T}} \in \mathbb{C}^L$ for each subcarrier $n = 0, ..., N-1$. The precoded symbols are transformed into the time-domain using N-point inverse fast Fourier transform (IFFT). Then a cyclic prefix (CP) whose length exceeds the delay spread of the channel is added before applying radio-frequency precoding. The g-th transmitted signal over subcarrier n is expressed as $\mathbf{F}^{(g)}[n]\mathbf{x}^{(g)}[n]$ where $\mathbf{F}[n] \in \mathbb{C}^{N_T \times L}$ is a beamforming matrix. In this paper, we consider the general beamforming matrix for a single user because considering specific beamformer is out of the scope.

C. Channel Model

We assume that the channel remains constant while the G symbols are transmitted. The $N_R \times N_T$ channel matrix $\mathbf{H}[n]$ for subcarrier n is given by [7]:

$$\mathbf{H}[n] = \mathbf{A}_R[n]\Gamma[n]\mathbf{A}_T^H[n]. \tag{1}$$

Here, $\mathbf{A}_R[n]$=$\mathbf{a}_{R,n}(\theta_R, \varphi_R)[n]$ and $\mathbf{A}_T[n]$=$\mathbf{a}_{T,n}(\theta_T, \varphi_T)[n]$ are the unit-norm array response vectors defined as follows:

$$\mathbf{a}_{R,n}(\theta_R, \varphi_R) = \frac{1}{\sqrt{N_R}} e^{-j\Delta_R^T \mathbf{s}_n(\theta_R, \varphi_R)}, \quad \in \mathbb{C}^{N_R} \tag{2}$$

$$\mathbf{a}_{T,n}(\theta_T, \varphi_T) = \frac{1}{\sqrt{N_T}} e^{-j\Delta_T^T \mathbf{s}_n(\theta_T, \varphi_T)}, \quad \in \mathbb{C}^{N_T} \tag{3}$$

where $\mathbf{s}_n(\theta, \varphi) = \frac{2\pi}{\lambda_n}[\sin\theta\cos\varphi, \sin\theta\sin\varphi]^T$ is the wavenumber vector with respect to the elevation angle θ and the azimuth angle φ, λ_n is the wavelength of the n-th subcarrier, Δ_R, and Δ_T are positions of the local Cartesian coordinates of the receive and transmit antenna elements. Denoting the complex channel gain, path loss, TOA, and sampling period by h, ρ, τ and T_s, $\Gamma[n]$ is expressed as

$$\Gamma[n] = \sqrt{N_R N_T} \frac{h}{\sqrt{\rho}} e^{-j2\pi n\tau/(NT_s)}. \tag{4}$$

After CP removal and FFT, the received signal over subcarrier n of transmission g is

$$\mathbf{y}^{(g)}[n] = \mathbf{H}[n]\mathbf{F}[n]\mathbf{x}^{(g)}[n] + \mathbf{n}^{(g)}[n] \tag{5}$$

where $\mathbf{n}^{(g)}[n] \in \mathbb{C}^{N_R}$ is a Gaussian noise vector with zero mean and variance $N_0/2$ per real dimension.

III. Unscented Kalman Filter for User Localization

A. Time Update

The user estimates the relative location from the UAV-BS and its rate of change using an UKF. The relative location is given by $\mathbf{r} = \mathbf{p}_U - \mathbf{p}_u$. Thus, the distance between the user and UAV-BS is denoted by $r = \|\mathbf{r}\|$. The state vector is defined as $\mathbf{x} = [\mathbf{r}^T, \dot{\mathbf{r}}^T]^T = [x_1, x_2, x_3, x_4]^T$. The state vector at time step $k+1$ is expressed as

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k - \mathbf{u}_{k,k+1} + \mathbf{G}_\mathbf{w}\mathbf{w}_k, \tag{6}$$

where $\mathbf{w} \sim \mathcal{N}(0, \mathbf{Q_r})$ is the process noise vector, $\mathbf{F}$ is the state transition matrix, $\mathbf{G_w}$ is the noise gain matrix, and $\mathbf{u}$ is the system input vector. These matrices and the vector are given by

$$\begin{gathered}\mathbf{F} = \begin{bmatrix} \mathbf{I} & \Delta t\mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \mathbf{G_w} = \begin{bmatrix} \frac{(\Delta t)^2}{2}\mathbf{I} \\ \Delta t\mathbf{I} \end{bmatrix}, \\ \mathbf{u}_{k,k+1} = [0, 0, \dot{x}_{u,k+1} - \dot{x}_{u,k}, \dot{y}_{u,k+1} - \dot{y}_{u,k}]^T, \end{gathered} \tag{7}$$

where $\mathbf{I}$ is the 2×2 identity matrix and Δt is the message interval between the user and the UAV-BS. Let $\bar{r}_0$, $\bar{\varphi}_{R,0}$, and $\bar{\varphi}_{u,0}$ are means of initial distance, AOA, and heading, respectively. In addition, σ_{r_0}, σ_{φ_R}, σ_{φ_u}, and σ_{v_u} are the standard deviations of r_0, φ_R, φ_u, and v_u, respectively. Using a Taylor series approximation, the initial mean state $\bar{\mathbf{x}}_0$ and initial covariance $\mathbf{P}_0$ are given by

$$\bar{\mathbf{x}}_0 = \begin{bmatrix} \bar{r}_0 \sin\bar{\varphi}_{R,0} \\ \bar{r}_0 \cos\bar{\varphi}_{R,0} \\ v_U \sin\varphi_U - v_u \sin\bar{\varphi}_{u,0} \\ v_U \cos\varphi_U - v_u \cos\bar{\varphi}_{u,0} \end{bmatrix}, \tag{8}$$

$$\begin{aligned} &\mathbf{P}_0 = \begin{bmatrix} \mathbf{P}_p & 0 \\ 0 & \mathbf{P}_v \end{bmatrix}, \quad \mathbf{T}(a,b) = \begin{bmatrix} \sin b & a\cos b \\ \cos b & -a\sin b \end{bmatrix}, \\ &\mathbf{P}_p = \mathbf{T}(\bar{r}_0, \bar{\varphi}_{R,0})\mathrm{diag}(\sigma_{r_0}^2, \sigma_{\varphi_R}^2)\mathbf{T}(\bar{r}_0, \bar{\varphi}_{R,0})^T, \\ &\mathbf{P}_v = \mathbf{T}(\bar{v}_u, \bar{\varphi}_{u,0})\mathrm{diag}(\sigma_{v_u}^2, \sigma_{\varphi_u}^2)\mathbf{T}(\bar{v}_u, \bar{\varphi}_{u,0})^T, \end{aligned} \tag{9}$$

The sigma points at step k are defined as

$$\chi_{i,k|k} = \begin{cases} \hat{\mathbf{x}}_{k|k} & i = 0 \\ \hat{\mathbf{x}}_{k|k} + (\sqrt{(\alpha+\kappa)\mathbf{P}_{k|k}})_i & i = 1, ..., \alpha \\ \hat{\mathbf{x}}_{k|k} - (\sqrt{(\alpha+\kappa)\mathbf{P}_{k|k}})_i & i = \alpha+1, ..., 2\alpha, \end{cases} \tag{10}$$

where α represents the dimension of $\mathbf{x}$ and κ is the design parameter. The weights W for sigma points are

$$W_i = \begin{cases} \frac{\kappa}{\alpha+\kappa} & i = 0 \\ \frac{1}{2(\alpha+\kappa)} & i = 1, ..., 2\alpha. \end{cases} \tag{11}$$

Then, at the $k+1$ time step, the estimated mean state $\hat{\mathbf{x}}_{k+1|k}$ and the covariance $\mathbf{P}_{k+1|k}$ propagated from $\mathbf{x}_k$ are as follows

$$\begin{aligned}\hat{\mathbf{x}}_{k+1|k} &= \sum_{i=0}^{2\alpha} W_i \bar{\chi}_{i,k+1}, \quad \bar{\chi}_{i,k+1} = \mathbf{F}\chi_{i,k|k} - \mathbf{u}_{k,k+1}, \\ \mathbf{P}_{k+1|k} &= \sum_{i=0}^{2\alpha} W_i (\bar{\chi}_{i,k+1} - \hat{\mathbf{x}}_{k+1|k})(\bar{\chi}_{i,k+1} - \hat{\mathbf{x}}_{k+1|k})^T \\ &\quad + \mathbf{G}_{\mathbf{w}}\mathbf{Q}_{\mathbf{r}}\mathbf{G}_{\mathbf{w}}^T.\end{aligned} \tag{12}$$

B. Measurement Update

From the observation (5), several techniques exist to recover the triplets of AOA, AOD and TOA as in [2], [3]. We assume that a channel estimation routine is presented to the user, which provides a set $\mathbf{z}_k$ of measurements at k time step. $\mathbf{z}_k = h(\mathbf{x}_k)$ can be expressed as

$$\mathbf{z}_k = [\varphi_{R,k}, \varphi_{T,k}, \tau_k]^T + \mathbf{v}_k \tag{13}$$

where $\mathbf{v} \sim N(0, \mathbf{R})$ is a measurement noise vector. The measurement noise covariance $\mathbf{R} = \mathrm{diag}(\sigma^2_{\varphi_R}, \sigma^2_{\varphi_T}, \sigma^2_r)$, where σ_{φ_T} and σ_r are standard deviations of AOD and distance. We can formulate the relationship between the channel parameters and the state of the system as follows:

$$\varphi_R = \arctan\frac{x_1}{x_2}, \quad \varphi_T = \frac{\pi}{2} - \varphi_R, \quad \tau = \frac{r}{c} \tag{14}$$

where c is the light speed (3×10^8 m/s). The estimated mean measurements and their covariances for computing the Kalman gain are defined as

$$\begin{aligned}&\xi_{i,k+1|k} = h(\chi_{i,k+1|k}), \quad \hat{\mathbf{z}}_{k+1|k} = \sum_{i=0}^{2\alpha} W_i \xi_{i,k+1|k} \\ &\mathbf{P}_{zz,k+1|k} = \sum_{i=0}^{2\alpha} W_i (\xi_{i,k+1|k} - \hat{\mathbf{z}}_{k+1|k})(\xi_{i,k+1|k} - \hat{\mathbf{z}}_{k+1|k})^T + \mathbf{R} \\ &\mathbf{P}_{xz,k+1|k} = \sum_{i=0}^{2\alpha} W_i (\chi_{i,k+1|k} - \hat{\mathbf{x}}_{k+1|k})(\xi_{i,k+1|k} - \hat{\mathbf{z}}_{k+1|k})^T.\end{aligned} \tag{15}$$

The Kalman gain $\mathbf{K}_{k+1}$, updated mean state and covariance are as follows:

$$\begin{aligned}&\mathbf{K}_{k+1} = \mathbf{P}_{xz,k+1|k}\mathbf{P}^{-1}_{zz,k+1|k} \\ &\hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}_{k+1}(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}) \\ &\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1}\mathbf{P}_{zz,k+1|k}\mathbf{K}^T_{k+1}.\end{aligned} \tag{16}$$

IV. Bearing Observability Maximization

For the localization errors to be bounded, the system is required to be observable. However, in the UAV-BS mmWave system, bearing observability cannot be guaranteed because of the movement of the UAV-BS. Fig. 2 shows the geometric relationship between the user and UAV-BS in two consecutive time steps, where the user and UAV-BS move from A_1 to A_2 and B_1 to B_2, respectively. If ϕ, the separation angle between $\mathbf{r}_k$ and $\mathbf{r}_{k+1}$, is not 0 and range from the UAV-BS is available to the user, the relative location vector $\mathbf{r}_{k+1}$ can be uniquely determined; in other words, the system is observable in bearing-based localization.

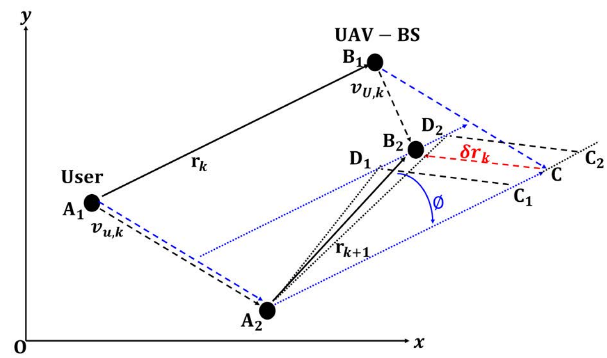

Fig. 2. Geometric relationship between the user and UAV-BS in two consecutive time instants. The blue line stands for the parallel translation of the relative location vector r_k to k+1 time step point.

To assist the user localization, the intervals for transmitted messages can be adjusted by the user's feedback to the UAV-BS. The user considers its bearing observability metrics at every time step and requests the UAV-BS to send its state message in the optimal message interval. In Fig. 2, by using the law of sines for the triangles $\triangle A_2D_1D_2$, $\triangle A_2C_2D_2$ and $\triangle A_2CB_2$, the relative location estimation error $\delta\varepsilon = D_1D_2$ is given by [8]

$$\delta\varepsilon^2 = \epsilon^2\left(\frac{\|\mathbf{r}_{k+1}\|}{\sin\phi}\right) \tag{17}$$

where ϵ is the bearing measurement error related to the number of antennas, its shape, and the channel estimation techniques. To minimize (17), the user calculates the optimal message interval Δt^* maximizing the objective function J:

$$J = \frac{\sin\phi}{\|\mathbf{r}_{k+1}\|} \tag{18}$$

where the separation angle ϕ is given by

$$\phi = \arccos\left(\frac{\mathbf{r}_k^T \cdot \mathbf{r}_{k+1}}{\|\mathbf{r}_k\|\|\mathbf{r}_{k+1}\|}\right). \tag{19}$$

The relative location vector at time step k+1 is given by

$$\begin{aligned}\mathbf{r}_{k+1} &= \mathbf{r}_k + \delta\mathbf{r}_k \\ &= [x_{1,k+1}, x_{2,k+1}] = [x_{1,k} + \delta x_{1,k}, x_{2,k} + \delta x_{2,k}]\end{aligned} \tag{20}$$

where $[\delta x_{1,k}, \delta x_{2,k}]$ is expressed as

$$[\delta x_{1,k}, \delta x_{2,k}] = [v_u\Delta t\cos\varphi_u - v_{U,x}, v_u\Delta t\sin\varphi_u - v_{U,y}]. \tag{21}$$

Here, $v_{U,x}$ and $v_{U,y}$ are the x-and y-components of v_U, respectively. By substituting (20) and (19) into (18), J^2 can be obtained by

$$J^2 = \frac{1-\cos^2\phi}{x^2_{1,k+1} + x^2_{2,k+1}} = \frac{(x_{1,k}x_{2,k+1} - x_{1,k+1}x_{2,k})^2}{(x^2_{1,k} + x^2_{2,k})(x^2_{1,k+1} + x^2_{2,k+1})^2}. \tag{22}$$

The partial derivative of (22) with respect to Δt provides a constraint for identifying the optimal message interval Δt^*,

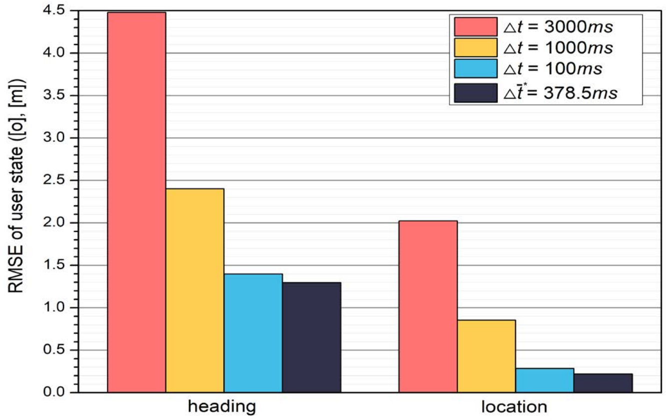

Fig. 3. RMSE of the user states according to the message intervals.

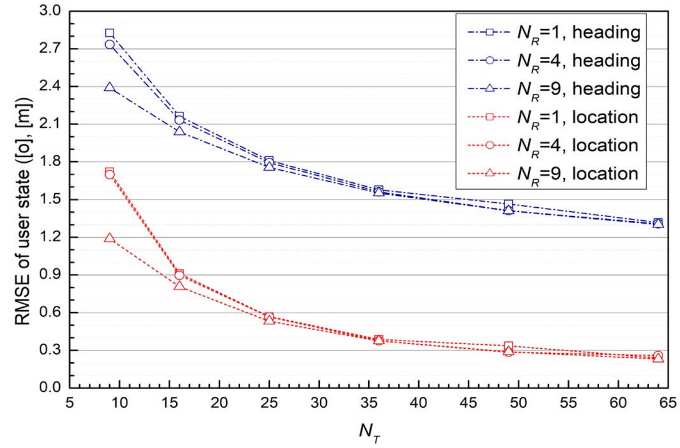

Fig. 4. RMSE of the user states according to the number of antennas.

i.e., $\partial J^2/\partial \Delta t = 0$, $\Delta t > 0$. The optimal message interval then yields:

$$\Delta t^* = \frac{x_{2,k} v_{U,x} - x_{1,k} v_{U,y}}{v_u (x_{2,k} \sin \varphi_u - x_{1,k} \cos \varphi_u)}. \tag{23}$$

If the optimal message interval cannot be identified, that is, $\Delta t^* \leq 0$, the user designates the same message interval as the past time step.

V. Simulation Results

We simulate 1,000 Monte Carlos events using MATLAB, each of which progresses over 100 time steps. The UAV-BS is assumed to fly at a constant velocity $v_U = 10$ m/s and constant turning rate $-\pi/60$ rad/s. The user moves in a straight line with a constant velocity $v_u = 20$ m/s and constant heading $\varphi_u = 2.444$ rad. The parameters for system initialization are $\varphi_{U,0} = -2.444$ rad, $\bar{r}_0 = 300$ m, $\sigma_{r_0} = 100$ m, $\varphi_{R,0} = 1.396$ rad, $\sigma_{\varphi_u} = 5$ rad, $\sigma_{v_u} = 1.028$ m/s. We consider the channel estimation parameters from (5) to be $\sigma_{\varphi_R} = 2\arcsin(0.891/N_R)$ rad, $\sigma_{\varphi_T} = 2\arcsin(0.891/N_T)$ rad, and $\sigma_r = 0.1$ m [9]. The parameters for UKF are $Q_{\mathbf{r}} = \mathrm{diag}(0.0256, 0.0256)$, $\alpha = 4$ and $\kappa = -1$.

In Fig. 3, the root-mean-square error (RMSE) performances of the user states, heading and location, are investigated. During the simulations, the number of antennas are set to $N_R = 9$ and $N_T = 64$. We evaluate RMSE with respect to the fixed message intervals $\Delta t = (3000, 1000, 100)$ ms and changeable optimal message interval with an expectation of 378.5 ms. It is observed that short message intervals generally have better RMSE performance; however, they require more communication resources. The proposed method requires a longer message interval, which requires fewer communication resources, and the RMSE performance is better. This is because the fixed message interval does not reflect the relative dynamics between the user and UAV-BS; hence, unobservable measurements decrease the filter efficiency.

Fig. 4 illustrates the RMSE performances according to the number of transmit and receive antennas. The optimal message interval, whose expectation is 342.4 ms, is applied. Due to the limitation of deploying a large number of antennas at the user side, we consider the RMSE performances versus the number of UAV-BS transmit antennas to consider the bearing measurement errors. It is observed that increasing the number of antennas on both sides results in better user localization. In particular, the proposed method provides sufficient performance while relatively few antennas are used.

VI. Conclusion

In this article, we propose a UAV-BS assisted localization method for mmWave systems by considering bearing observability. The UKF based estimation method and optimal message time interval are investigated to consider the geometric relationship based bearing observability. The simulation results demonstrate the effectiveness of the proposed method.

References

[1] F. Wen, H. Wymeersch, B. Peng, W. P. Tay, H. C. So, and D. Yang, "A survey on 5G massive mimo localization," *Digital Signal Processing*, vol. 94, pp. 21–28, 2019.

[2] A. Shahmansoori, G. E. Garcia, G. Destino, G. Seco-Granados, and H. Wymeersch, "Position and orientation estimation through millimeter-wave mimo in 5G systems," *IEEE Transactions on Wireless Communications*, vol. 17, no. 3, pp. 1822–1835, 2017.

[3] Z. Lin, T. Lv, and P. T. Mathiopoulos, "3-D indoor positioning for millimeter-wave massive mimo systems," *IEEE Transactions on Communications*, vol. 66, no. 6, pp. 2472–2486, 2018.

[4] Z. Xiao, P. Xia, and X.-G. Xia, "Enabling UAV cellular with millimeter-wave communication: Potentials and approaches," *IEEE Communications Magazine*, vol. 54, no. 5, pp. 66–73, 2016.

[5] S. Büyükçorak, G. K. Kurt, and A. Yongaçoğlu, "UAV assisted ground user localization," in *2019 IEEE International Conference on Wireless for Space and Extreme Environments (WiSEE)*. IEEE, 2019, pp. 111–115.

[6] J. Li, Z. Ning, S. He, C.-H. Lee, and S. Zhao, "Three-dimensional bearing-only target following via observability-enhanced helical guidance," *IEEE Transactions on Robotics*, vol. 39, no. 2, pp. 1509–1526, 2022.

[7] R. W. Heath, N. Gonzalez-Prelcic, S. Rangan, W. Roh, and A. M. Sayeed, "An overview of signal processing techniques for millimeter wave MIMO systems," *IEEE J. Sel. Top. Signal Process.*, vol. 10, no. 3, pp. 436–453, Apr. 2016.

[8] S. He, H.-S. Shin, and A. Tsourdos, "Trajectory optimization for target localization with bearing-only measurement," *IEEE Transactions on Robotics*, vol. 35, no. 3, pp. 653–668, 2019.

[9] Z. Abu-Shaban, X. Zhou, T. Abhayapala, G. Seco-Granados, and H. Wymeersch, "Error bounds for uplink and downlink 3D localization in 5G millimeter wave systems," *IEEE Transactions on Wireless Communications*, vol. 17, no. 8, pp. 4939–4954, 2018.