Antenna Selection Optimization in Polarization Reconfigurable MIMO System

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***Abstract*—By selecting the best M' out of M antennas, antenna selection significantly lowers the hardware complexity of MIMO system with large antenna elements at both link ends. The computational complexity of the widely used antenna selection method is NP-hard. We lower the complexity by formulating the selection problem as a convex optimization problem. We apply this technique to the polarization reconfigurable MIMO (PR-MIMO) system to additionally gain the benefit of polarization diversity to the system. Our simulation result validates that by employing antenna selection to PR-MIMO, we can achieve full conventional uni-polarized MIMO capacity with just a subset of antennas. Further, using convex optimization for antenna selection gives a close agreement to that obtained by brute-force numerical search, while yielding lower computational complexity.**

***Index Terms*—Antenna Selection, Polarization, Convex Optimization**

I. Introduction

Multiple input multiple output (MIMO) communication system has been the furnace of wireless communication system for the past recent years. Naturally, much effort has been invested by the leading scholars to enhance the MIMO system. One particular area for enhancement has been to address the high hardware complexity of MIMO which require $N \times M$ number of RF chains for a system with N number transmitters (Tx) and M number of receivers (Rx). The two main methods for exploiting MIMO systems are spatial multiplexing, and diversity. In the former case, independent data streams are transmitted and received through spatial parallelized MIMO channels. The parallelization is created via precoding and postcoding based on singular value decomposition (SVD) of the channel matrix. Another way to exploit MIMO system is to utilize Tx and Rx diversity purely for link-quality improvement. Tx antennas employ maximal-ratio transmission (MRT) to simultaneously transmit weighted replica of the single bit stream. The ideal weights are obtained by SVD of the channel impulse matrix. In the similar manner, Nr Rx antennas utilize maximal-ratio combining (MRC), where the weighted received signals are linearly combined to increase the effective SNR. Overall, a diversity degree of Nt $\cdot$ Nr can be obtained.

The structure of MIMO system is inherently prone to hardware complexity because it requires $M \times N$ number of RF chains for a system with N number of transmitters (Tx) and M number of receivers (RX). MIMO is exposed to hardware complexity when number of antenna elements becomes large, because hardware complexity increases proportionally to number antenna elements. The state-of-the-art wireless communication system is trending towards increasing the antenna elements as explained in [1]. For example, fifth generation (5G) new radio (NR) base stations, Node B (gNB), has deployed antenna panels that contain at least 64 elements in a single panel as described in. Therefore, the need for a system with lower hardware complexity is desired. In this context, antenna selection is a promising choice that can significantly mitigate this problem because it chooses a subset of antenna elements which captures a large portion of the full MIMO system capacity. By selecting M' out of M Rx antennas, antenna selection reduces the hardware complexity by lowering the number of RF chains of Rx from M to M'.

Another way to enhance MIMO system is to incorporate polarization diversity. Polarization diversity has demonstrated a promising potential to improve MIMO system in terms of symbol error rate (SER) and channel capacity. In particular, [2] describes a polarized-MIMO system that significantly increases the channel capacity from that of the conventional MIMO system. To further enhance the MIMO system, this paper serves to combine polarization diversity and antenna selection by capturing the benefit they both provide.

The advantage of antenna selection is demonstrated in [3], [4]. However, polarization diversity is not taken into account in the majority of previous research works. Although there are previous reports that consider polarization diversity with antenna selection, they consider fixed antenna polarization, [5], [6]. In contrast, we exploit antenna selection with polarization-agile antenna elements which significantly outperforms the conventional scheme of the conventional MIMO system. Further, most of the antenna selection algorithm has high computational complexity, as the antennas are selected with brute-force search. However, this paper formulates and solves the antenna selection problem as a convex optimization problem which yields lower computational complexity.

II. System Model

Polarization reconfigurable MIMO (PR-MIMO) system with antenna selection is illustrated by Fig. 1, where antenna elements change the antenna polarization angles to any continuous degrees. Our objective is to select M' out of M such antenna

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elements at Rx. The effective channel matrix of PR-MIMO system is described as

$$\mathbf{H}^{\text{eff}} = \begin{bmatrix} \vec{p}_{\text{Rx},1}^{\,T} H_{11} \vec{p}_{\text{Tx},1} & \dots & \vec{p}_{\text{Rx},1}^{\,T} H_{1N} \vec{p}_{\text{Tx},N} \\ \vdots & \ddots & \vdots \\ \vec{p}_{\text{Rx},M}^{\,T} H_{M1} \vec{p}_{\text{Tx},1} & \dots & \vec{p}_{\text{Rx},M}^{\,T} H_{MN} \vec{p}_{\text{Tx},N} \end{bmatrix}, \quad (1)$$

where the operation $(\cdot)^T$ is the transpose of a given vector or matrix. Further, H_{ij} is called "*polarization-basis matrix*", which is expressed as

$$H_{ij} = \begin{bmatrix} h_{ij}^{\text{vv}} & h_{ij}^{\text{vh}} \\ h_{ij}^{\text{hv}} & h_{ij}^{\text{hh}} \end{bmatrix}, \quad (2)$$

where h_{ij}^{xy} with $\text{x} \in \{\text{v}, \text{h}\}$; $\text{y} \in \{\text{v}, \text{h}\}$ is the XY-channel impulse response from the Y-polarization Tx antenna to the X-polarization Rx antenna. Each entry of (2) is modeled as independent identically distributed (i.i.d.) zero-mean, circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. Lastly, $\vec{p}_{\text{Tx},j}$ and $\vec{p}_{\text{Rx},i}$ are, respectively, the Tx-polarization vector at the jth Tx antenna and the Rx-polarization vector at the ith Rx antenna, and they are expressed as

$$\vec{p}_{\text{Tx},j} = \begin{bmatrix} p_{\text{Tx},j}^{\text{v}} \\ p_{\text{Tx},j}^{\text{h}} \end{bmatrix} = \begin{bmatrix} \cos\theta_j \\ \sin\theta_j \end{bmatrix}, \quad (3)$$

$$\vec{p}_{\text{Rx},i} = \begin{bmatrix} p_{\text{Rx},i}^{\text{v}} \\ p_{\text{Rx},i}^{\text{h}} \end{bmatrix} = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}. \quad (4)$$

Here, we call the angles θ_j and θ_i Tx- and Rx-polarization angles, respectively. It is worth mentioning that Tx- and Rx-polarization vectors are unit vectors so that the overall signal power is preserved. Optimal polarization vectors that maximize the sum of squared singular value of (1) is described in detail in [2].

We consider a system where Tx does not know about the channel while Rx does. Then the capacity of PR-MIMO system is described by

$$C(\mathbf{H}^{\text{eff}}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{R}_{\mathbf{ss}} (\mathbf{H}^{\text{eff}})^H \mathbf{H}^{\text{eff}}), \quad (5)$$

where γ is the signal to noise ratio (SNR), $\mathbf{I}_N$ is the $N \times N$ identity matrix and $\mathbf{R}_{\mathbf{ss}}$ is the covariance matrix of Tx signals. Since Tx does not know about the channel, $\mathbf{R}_{\mathbf{ss}}$ is chosen as I_N/N and optimal polarization is found only at the Rx while the Tx has its antenna polarization at random angles.

III. Antenna Selection as Convex Optimization Problem

Antenna selection chooses M' out of M receivers. We express (5) with M' selected receivers as

$$C_r(\mathbf{H}_{\mathbf{r}}^{\text{eff}}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{R}_{ss} (\mathbf{H}_{\mathbf{r}}^{\text{eff}})^H \mathbf{H}_{\mathbf{r}}^{\text{eff}}), \quad (6)$$

where the dimension of $\mathbf{H}_{\mathbf{r}}^{\text{eff}}$ is $M' \times N$. Further, we define (5) as a function of selected antennas by defining Δ_i as

$$\Delta_i = \begin{cases} 1, & \text{if } i^{th} \text{ receive antenna selected} \\ 0, & \text{if otherwise.} \end{cases} \quad (7)$$

Using (7), capacity described by (6) becomes a function of $\mathbf{\Delta}$ as

$$C(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H}^{\text{eff}} (\mathbf{H}^{\text{eff}})^H), \quad (8)$$

where $\mathbf{\Delta}$ is a diagonal matrix consists of Δ_i's. This is derived rigorously in [7].

Hence, we formulate the antenna selection problem as

$$\begin{aligned} \max \quad & \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H}^{\text{eff}} (\mathbf{H}^{\text{eff}})^H) \\ \text{subject to} \quad & \Delta_i = \{0, 1\}, \\ & \text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'. \end{aligned} \quad (9)$$

The objective is to find Δ_i's which maximize (6). We observe that this problem is NP hard because it is solved with brute-force search with $\binom{M}{M'}$ cardinality as described in [8]. We seek to formulate the problem into a simpler problem by applying relaxation on the Δ_i's by allowing $\Delta_i \in [0, 1]$. This problem then becomes a convex optimization problem with lower complexity. The reformulated problem is described as follow

$$\begin{aligned} \max \quad & \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H}^{\text{eff}} (\mathbf{H}^{\text{eff}})^H) \\ \text{subject to} \quad & 0 \le \Delta_i \le 1, \quad i = 1, ..., M, \\ & \text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'. \end{aligned} \quad (10)$$

We apply a rounding scheme after the solution is found, where we round the highest M' Δ_i's to 1 and the rest to 0; which indicates the selected antenna. This is solved efficiently using the CVX solver [9]. We compare the capacity of PR-MIMO to conventional MIMO. It is worth to note that conventional MIMO employ $M \times N$ channel matrix $\mathbf{H}$ whose entries are (ZMCSCG); therefore, the capacity of convention MIMO system can be analyzed by replacing $\mathbf{H}^{\text{eff}}$ by $\mathbf{H}$ in (5) and (10).

IV. Experiments

In this section, we present the performance of our system with experiment results found via Monte-Carlo simulation. We obtain the average capacity of over 2000 realization of the channel matrix for SNR regime from 0 to 20 dB. The result is illustrated in Fig. 2. The simulation parameters are as follows, $N = 2$, $M = 6$ and $M' = 2$. We computed the capacity of conventional MIMO system, using $\mathbf{H}$ as channel matrix, for various scenarios: when M antennas are used (blue curve), 2 optimally selected antennas with (10) are used (red curve), 2 selected antennas with brute-force search (green curve) are used and 2 randomly selected antennas are used (yellow curve). We compare these result with 2 selected antennas in PR-MIMO system (black curve) found by (10). Fig. 2 conveys that optimally selected antennas (red) yield higher capacity than that of randomly selected antennas (yellow). Moreover, its capacity has very close agreement to that of the capacity of selected antennas found with brute-force search (green). This proves that the convex method performs as well as the brute-force method. By applying (10) to the PR-MIMO system (black),

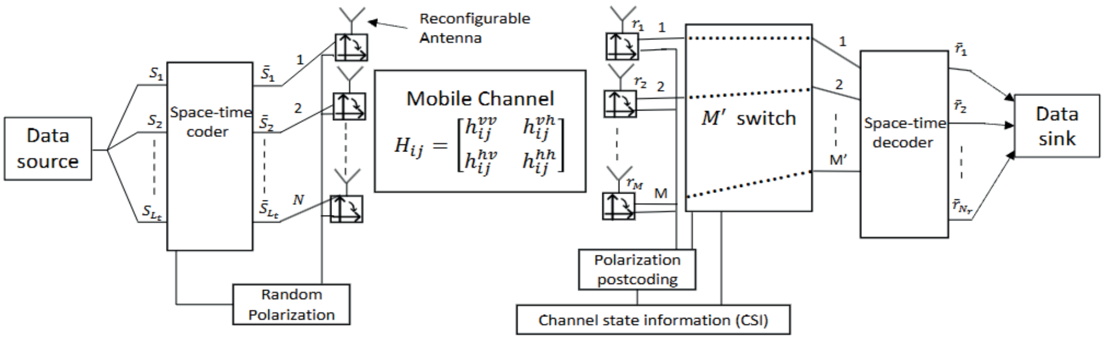

Fig. 1: Antenna selection in PR-MIMO system.

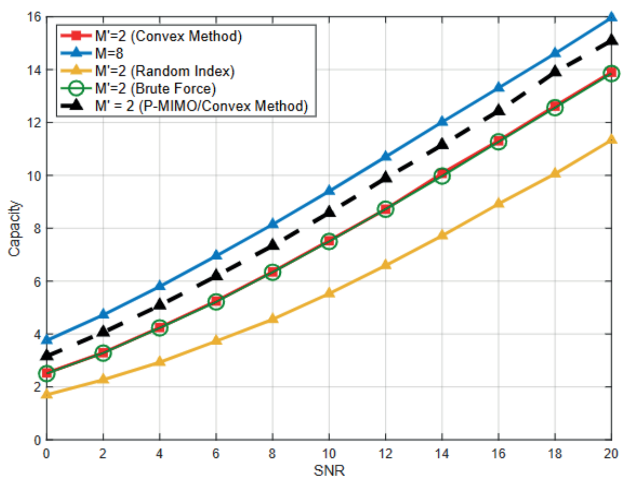

Fig. 2: Capacity v/s SNR, $M = 6$, $N = 2$, $M' = N$

the capacity is close to the capacity found with full antennas of conventional MIMO system (blue); therefore, exhibiting that PR-MIMO combined with antenna selection further enhanced antenna selection system from that of conventional MIMO system.

V. Discussion

The exhaustive-search selector evaluates $\binom{M}{M'}$ subsets, and each capacity evaluation requires a Cholesky factorization of an $M \times M$ matrix, i.e., $\Theta\big(\binom{M}{M'} M^3\big)$ flops overall. In contrast, the relaxed problem in (10) is convex and we solve it with CVX (interior-point). Each iteration then costs $O(MN^2)$ plus $O(N^3)$. Theoretical iteration complexity is $O(\sqrt{M}\log(1/\varepsilon))$ [10], while in practice 20–40 iterations suffice. Hence the total work scales as $\tilde{O}\big(\sqrt{M}\,(MN^2 + N^3)\big)$ in worst case and roughly $O(MN^2 + N^3)$ empirically, which is polynomial and dramatically smaller than the combinatorial exhaustive search. The final rounding step to pick the best M' antennas is $O(M \log M)$.

VI. Conclusion

This paper finds the best subset of polarization-agile antennas of PR-MIMO system which captures the large portion of the full system capacity. Antenna selection problem is formulated into a convex optimization problem which was solved efficiently using CVX Solver. The result shows that the proposed method yield a capacity that has a very close agreement with the capacity found with brute-force search; showing that we have the advantage over the brute force method because our method has lower complexity.

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