An Inhomogeneous Plane Wave-based Ray Tracing Algorithm for LEO Atmospheric Propagation Modeling

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***Abstract*—In this conference paper, we introduce a ray tracing algorithm for low Earth orbit satellite communications where an inhomogeneous plane wave propagates. We analyze our approach in terms of its physical meaning founded on the well-known ray tracing methods in lossy media that is previously proposed. We organize some mathematical formulations using the complex Snell's law and incorporate the two numerical methods.**

***Keywords*—ray tracing, inhomogeneous plane wave, low Earth orbit satellite communications, geometrical optics, complex Snell's law, atmospheric refractive index.**

I. Introduction

In recent years, there has been growing interest on leveraging the low Earth orbit (LEO) satellites – the essential systems for implementing the non-terrestrial networks (NTNs) – to provide seamless communication services to end-users, particularly in research area on 5G wireless communication systems. Furthermore, the LEO satellites offer key advantages in realizing low latency, extensive coverage for the upcoming next-generation wireless communication systems [1]. Electro-magnetic waves radiated from the transmit antennas mounted on the LEO satellites are subject to atmospheric refraction and attenuation. These atmospheric phenomena can degrade the performance of the communication, thereby reducing the quality of service (QoS). Therefore, to mitigate the impact of the atmosphere on the communications and to ensure an efficient, optimized operation of the LEO-based systems, it is necessary to develop a model how the electromagnetic waves propagate and reach to the receiver.

Due to the phenomena of refraction and attenuation when the wave propagates, the atmosphere of the Earth can be characterized as a lossy medium, with the extent of these effects governed by its complex-valued atmospheric refractive index. This refractive index is determined by meteorological factors such as pressure, temperature, relative humidity and waterfall rate [2,3]. In other words, the atmospheric property varies with altitude and horizontal space. This observation allows the atmosphere to be viewed as a stratified structure, which can be divided into numerous layers based on those varying characteristics. With this understanding, a ray tracing is considered as a suitable numerical analysis method to describe the propagation of the electromagnetic waves, given the large spatial scale of the atmospheric domain [4]. However, to the best of our knowledge, there has been no rigorous investigation into their underlying mathematical formulations nor any detailed description of the ray tracing algorithm for inhomogeneous plane wave.

Radcliff and Balanis established the theoretical basis for the propagation of inhomogeneous plane wave in lossy media [5]. The key concept of their approach was to modify the propagation constants to avoid intricacy when solving the complex Snell's law, while maintaining the physical interpretability of the wave behavior. Chang and their colleagues proposed a ray tracing method in lossy media by considering a relationship between the complex wave vector and the refractive index [6]. Chang's pioneering work on the complex ray tracing has served as a foundational reference, with many subsequent studies citing and building upon it on applying the method to their various research [7,8,9].

However, to the best of our understanding, while Chang's work presents theoretical insights, it does not appear to provide detailed explanations of the ray tracing algorithm itself, and raises the possibility that the physical interpretation of wave refraction might not be fully preserved—particularly in our study on the Earth's atmospheric refractivity. Moreover, many research works adopt the mathematical representations formulated by Chang's work almost unchanged and without providing any explanation as to the reasoning behind its derivation. In contrast, Ballington and Hesse provided a derivation of the propagation direction formula—originally presented in Chang's work—in the appendix of their study on light scattering by large particles, in which they also introduced a *backward* ray tracing algorithm as opposed to the forward ray tracing method [7].

In this work, we provide detailed derivations of the mathematical representations that describe wave refraction, incorporating both Radcliff's and Chang's approaches. Furthermore, we present the results of applying the ray tracing algorithm to an inhomogeneous plane wave propagation in a lossy medium with a simple example. Specifically, we introduce 2x2 grid structure in a two-dimensional Cartesian coordinate system and assume a ray propagation within a medium whose refractive index includes an imaginary component on the order comparable to that of atmospheric refractive index [10]. Here, we assume that each 2D square-shaped stratified structure is an isotropic, linear, charge-free, and homogeneous medium. The time-harmonic convention used throughout this work is $e^{j\omega t}$. The tilde indicates physical quantities that are complex-valued and boldface denotes vector quantities.

II. Fundamentals and Derivations

A. Characteristic Relations in a Lossy Medium

A lossy medium can be characterized with the complex refractive index $\tilde{n} = n - jk$, which is related to the relative permittivity as shown below, revealing that the permittivity takes a complex form,

$$\tilde{\varepsilon} = \varepsilon_r \varepsilon_0 - j\frac{\sigma}{\omega} = \varepsilon_0 \tilde{n}^2 \quad (1)$$

Wave vector $\widetilde{\boldsymbol{k}}$ is represented as a complex quantity in lossy media, allowing the separation of the direction of phase propagation and the direction of attenuation. These directions correspond to vectors normal to the surfaces of constant phase $\bar{\boldsymbol{e}}$ and constant amplitude $\bar{\boldsymbol{f}}$, respectively. A plane wave in which these two vectors are not aligned is called an inhomogeneous plane wave. When a homogeneous plane wave impinges on a lossy medium, it propagates as an inhomogeneous plane wave within the medium.

$$\widetilde{\boldsymbol{k}} = N\bar{\boldsymbol{e}} - jK\bar{\boldsymbol{f}}, \quad (2)$$

where N and K are apparent refractive indices [6]. Complex-valued atmospheric refractivity $\widetilde{N}$ and its associated atmospheric refractive index $\tilde{n}$ are expressed by following equation (3). This implies that the atmosphere of the earth can be modeled as a lossy medium and that wave propagation occurs in the form of inhomogeneous plane waves at altitudes where the meteorological phenomena take place,

$$\tilde{n} = 1 + \widetilde{N} \times 10^{-6}\ [\mathrm{ppm}]. \quad (3)$$

From studies on atmospheric refractive-index modeling [10], we found that the imaginary part of the complex refractive index is on the order of 10^{-7} or smaller. Based on this magnitude, we implemented a ray tracing algorithm that accounts for absorption.

To describe propagation of the inhomogeneous plane wave under the framework of geometrical optics, the propagation constant must be determined. Following Radcliff's previous work, intrinsic propagation constant of each medium, γ_0 is computed from its electrical conductivity σ_{k}, angular frequency ω, and the complex relative permittivity $\tilde{\varepsilon}_{\mathrm{k}}$ as shown below, for the case where two media form a planar interface.

$$\gamma_{0\mathrm{k}} = \sqrt{(\sigma_{\mathrm{k}} + j\omega\tilde{\varepsilon}_{\mathrm{k}})(j\omega\mu_0)} = \alpha_{0\mathrm{k}} + j\beta_{0\mathrm{k}}, \quad (4)$$

where k = 1, 2 and $\boldsymbol{\gamma}_{0\mathrm{k}} = \mathrm{j}\widetilde{\boldsymbol{k}}$. Equations (1), (3) and (4) suggest that, once the atmospheric refractivity is evaluated, the electrical conductivity and complex relative permittivity of each subdivided square-shaped stratified structure can be obtained. These parameters yield the intrinsic propagation constant of each medium, which subsequently allows for the computation of the modified propagation constant introduced in Radcliff's work.

B. Modified Propagation Constants

Snell's law is derived by applying the phase-matching condition at a planar interface. Consequently, it yields two separate expressions: one for the propagation constant α_{k}, which accounts for attenuation of the wave, and another for the propagation constant β_{k}, which governs the direction of wave propagation.

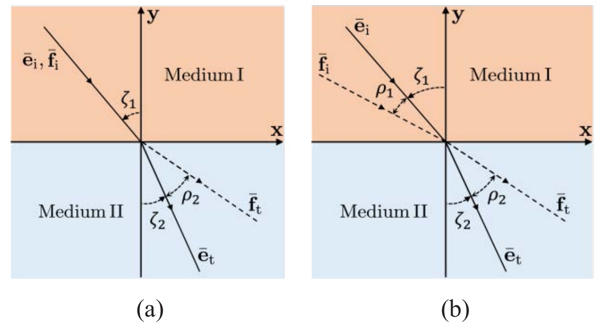


Fig. 1. Illustration of ray propagation at a planar interface: (a) transition from a lossless to a lossy medium, and (b) propagation behavior within the lossy medium. For clarity, only the incident and transmitted rays are shown. A homogeneous plane wave, incident from a lossless medium, becomes an inhomogeneous plane wave upon encountering a lossy medium.

$$\alpha_{01}\sin(\zeta_1 + \rho_1) = \alpha_2 \sin(\zeta_2 + \rho_2), \quad (5)$$

$$\beta_{01}\sin\zeta_1 = \beta_2 \sin\zeta_2, \quad (6)$$

$$\alpha_2^2 - \beta_2^2 = \alpha_{02}^2 - \beta_{02}^2, \quad (7)$$

$$\alpha_2\beta_2\cos\rho_2 = \alpha_{02}\beta_{02}, \quad (8)$$

where $\gamma_{\mathrm{k}} = \alpha_{\mathrm{k}} + j\beta_{\mathrm{k}}$ and γ_{k} denotes the modified propagation constant.

$$\alpha_1 = \alpha_{01}\sin(\zeta_1 + \rho_1), \quad (9)$$

$$\beta_1 = \beta_{01}\sin\zeta_1. \quad (10)$$

By applying Equation (9), the equation (5) can be rewritten in an alternative form; similarly, the equation (6) is reformulated using Equation (10), as follows.

$$\sin(\zeta_2 + \rho_2) = \frac{\alpha_1}{\alpha_2}, \quad (11)$$

$$\sin\zeta_2 = \frac{\beta_1}{\beta_2}, \quad (12)$$

$$\cos(\zeta_2 + \rho_2) = \sqrt{1 - \sin^2(\zeta_2 + \rho_2)}, \quad (13)$$

$$\cos\zeta_2 = \sqrt{1 - \sin^2\zeta_2}. \quad (14)$$

Substituting equations (11) through (14) into equation (8) and applying trigonometric identities, a relationship is then derived between the modified propagation constants in the two media and the intrinsic propagation constant of the second (transmitting) medium as follows,

$$(\alpha_1\beta_1 - \alpha_{02}\beta_{02})^2 = (\alpha_2^2 - \alpha_1^2)(\beta_2^2 - \beta_1^2). \quad (15)$$

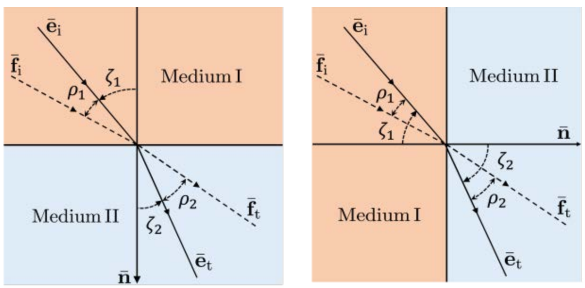
A quartic equation form for α_2 emerges by substituting equation (16) into equation (7) as below,

$$\alpha_2^4 - [\alpha_1^2 + \beta_1^2 + \alpha_{02}^2 - \beta_{02}^2]\alpha_2^2 + \alpha_1^2\beta_1^2 + (\alpha_{02}^2 - \beta_{02}^2)\alpha_1^2 - (\alpha_{02}\beta_{02} - \alpha_1\beta_1)^2 = 0. \tag{17}$$

By replacing α_2^2 with X, the quartic equation is transformed into a quadratic form, allowing the application of the discriminant method to derive an explicit expression for α_2^2,

$$X^2 - bX + c = 0, \tag{18}$$

where $b = \alpha_1^2 + \beta_1^2 + \alpha_{02}^2 - \beta_{02}^2$ and $c = \alpha_1^2\beta_1^2 + (\alpha_{02}^2 - \beta_{02}^2)\alpha_1^2 - (\alpha_{02}\beta_{02} - \alpha_1\beta_1)^2$ and the results for modified propagation constants in transmitting medium are consistent with those derived by Radcliff. That is, equations (18) and (19) in Radcliff's work can be interpreted as solutions to a nonlinear system derived from a set of four coupled first-order equations.



(a) (b)

Fig. 2. Illustrations for our simulation scenario. The phase vector $\bar{\boldsymbol{e}}$ and attenuation vector $\bar{\boldsymbol{f}}$ can be represented based on the previously derived refraction angles ζ_2, ρ_2. Each medium is distinguished by its planar planes such as x or y-plane. $\bar{n}$ stands for unit vector normal to the planar interface. Subscription of the vectors means incident ray and transmitted ray, respectively.

III. Ray Tracing Method

A. Scenario Descriptions and Algorithm

In a two-dimensional (2D) scenario, we assume that the atmosphere can be discretized into square-shaped small cells, and the electric field is polarized in the TE polarization. This setup is chosen to examine the ray refraction in the most intuitive manner, while the validity of the approach remains valid regardless of the polarization. Under these scenario, the planar interfaces of the media can be aligned with either the x- or y-plane. Therefore, we classify the analysis into two cases, depending on whether the electric field intersects the x-plane or the y-plane. This classification allows us to define vectors normal to the interface accordingly, enabling the phase and attenuation vectors to be expressed in terms of the refraction angle.

$$\zeta_2 = \sin^{-1}\left(\frac{\beta_{01}}{\beta_2}\sin\zeta_1\right), \tag{19}$$

$$\rho_2 = \sin^{-1}\left[\frac{\alpha_{01}}{\alpha_2}\sin(\zeta_1 + \rho_1)\right] - \zeta_2. \tag{20}$$

- Case 1: Ray intersects with the x-plane (vertical)

$$\bar{e}_t = \pm\cos\zeta_2\,\hat{x} + \sin\zeta_2\,\hat{y}, \tag{21}$$

$$\bar{f}_t = \pm\cos(\zeta_2 + \rho_2)\,\hat{x} + \sin(\zeta_2 + \rho_2)\,\hat{y}. \tag{22}$$

- Case 2: Ray intersects with the y-plane (horizontal)

$$\bar{e}_t = \sin\zeta_2\,\hat{x} \pm \cos\zeta_2\,\hat{y}, \tag{23}$$

$$\bar{f}_t = \sin(\zeta_2 + \rho_2)\,\hat{x} \pm \cos(\zeta_2 + \rho_2)\,\hat{y}, \tag{24}$$

where the sign preceding the cosine term is determined by the propagation direction of the incident ray.

Algorithm 1 Ray Tracing in Atmospheric Environment

input: atmospheric refractive index of two media $\tilde{n}_1$, $\tilde{n}_2$, origin P of ray, direction vector of incident ray $\bar{\boldsymbol{e}}_{\mathrm{i}}, \bar{\boldsymbol{f}}_{\mathrm{i}}$, initial electric field $\boldsymbol{E}^{(1)}$

output: direction vector $\bar{\boldsymbol{e}}_{\mathrm{i}}, \bar{\boldsymbol{f}}_{\mathrm{i}}$, electric field $\boldsymbol{E}^{(n+1)}$, arrival point A of ray

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calculate: properties of the stratified atmosphere such as conductivity σ, relative permittivity ε_r
while rays exist do
    set threshold to amplitude of updated E field
    if no rays to trace or |E^(n+1)| < threshold
        break
    end if
    set ray's initial information : origin P, direction of phase and attenuation ē_i, f̄_i, and electric field E^(n)
    find intersection point given direction and origin of incident ray
    calculate the modified propagation constant γ_k and refraction angle ζ_2 and ρ_2 by the complex Snell's law
    determine direction vectors of transmitted ray ē_t, f̄_t , E^(n+1) with attenuation and phase delay
    update ē_i ← ē_t, f̄_i ← f̄_t, E^(n+1) ← E^(n), P ← A
end while
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IV. Result and Concluding Remarks

In this work, we provide detailed derivations of the methods introduced by previous researchers and incorporate them to apply ray tracing method to the Earth's atmospheric environment, which is characterized by complex refractive index with a negligible imaginary component. The ray originates from a homogeneous plane wave under the assumption that the stratospheric medium is lossless. This is justified by the fact that, at higher altitudes, meteorological phenomena are minimal and the atmosphere can be approximated as a near vacuum due to the trivial presence of water vapor and gas molecules.

As shown in figure 3, when σ_2 increases toward value of σ_1, the conductivity contrast between the two media vanishes, causing the inhomogeneity of the transmitted ray to diminish and approach to zero. Conversely, when σ_2 falls below 10^{-5}, ρ_2 converges toward 90° and saturates, signifying the situation in which the conductivity of medium 1 vastly exceeds that of medium 2 and thus no attenuation occurs along the phase direction. When the value for medium 2 exceeds that for medium 1, inhomogeneity reappears, its magnitude initially increasing before decreasing. This corresponds to attenuation along the propagation direction gradually increasing, and when σ_2 increases to near 10^{-1}, as the plane wave restore its homogeneity, attenuation in the propagation direction reaches its maximum.

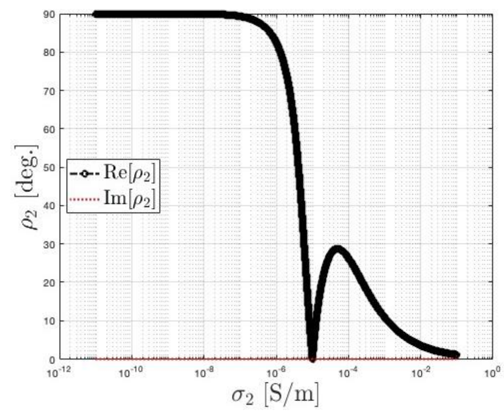

Fig. 3. This figure depicts ρ_2—decomposed into its real and imaginary parts and plotted in degrees—as σ_2 is varied for two media in contact at a planar interface, with σ_1 held constant as 10^{-5}. Here, σ_1 stands for electric conductivity of medium 1 whereas medium 2 has electric conductivity σ_2.

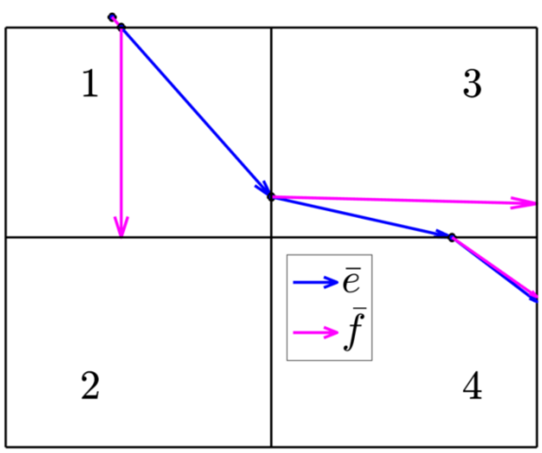

Fig. 4. Example illustration of inhomogeneous plane wave propagation for simple structures using ray tracing algorithm. A homogeneous plane wave becomes inhomogeneous when it propagates through a lossy medium. The numbers shown in this figure correspond to distinct media and refractive index of each media is $1 + 10^{-8}$, $2 + 2 \times 10^{-8}$, $3 + 3 \times 10^{-7}$ and $4 + 4 \times 10^{-7}$, respectively.

The homogeneity of the rays breaks immediately upon entering into the lossy medium, forming an inhomogeneous plane wave as a result in figure 4. This phenomenon is well-established in prior studies. Moreover, according to the complex Snell's law, the attenuation vector becomes aligned with the normal vector at the planar interface of the medium, which is consistent with the findings in [11, 12]. In addition, the complex Snell's law links the transmitted attenuation vector to the previous angle of incidence encountered at the interface. Therefore, as the wave continues to propagate, this causes the attenuation vector to roughly follow the direction of the phase vector. Then, the plane wave maintains its inhomogeneity as it propagates through the medium. When the ray propagates from medium 1 into medium 3, the imaginary part of the refractive index differs by a factor of ten, and by Snell's law this causes the angle between the attenuation vector and the interface normal to be small. Subsequently, as the ray enters medium 4 from medium 3—where the imaginary parts of the refractive indices are of the same order—the angle between the attenuation vector and the normal increases during propagation.

Given that earth's atmosphere can be regarded as a lossy medium, such inhomogeneous propagation may contribute to attenuation and boresight errors in using LEO satellites to communication systems. Future work will incorporate the ray tracing algorithm into a larger number of stratified structures to model wave propagation in the atmospheric environment with greater precision. To enable the development of efficient LEO satellite communication systems, this involves leveraging atmospheric refractivity modeling techniques such as MPM93 to perform high-resolution interpolation and prediction of refractive index profiles, thereby enabling accurate modeling of a more finely layered atmosphere.

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