# Tackling Traveling Salesman Problems with a Cluster-Based Quantum-Classical Optimization

Saweed Hussain†, Lutfiana Sausan†, Abdur Rahman†, Chimeremma Sandra Amadi† and Hoon Ryu†*
†Kumoh National Institute of Technology, Gumi 39177, Republic of Korea
*Email: elec1020@kumoh.ac.kr

***Abstract*—A hybrid quantum-classical framework is exploited to solve the Traveling Salesman Problem (TSP) based on a Quantum Approximate Optimization Algorithm (QAOA). To overcome the computational limitation of the Grover's Search Algorithm (GSA) stemming from path encoding and arithmetic subroutines, a clustering-based strategy is employed to decompose a 16-city TSP into four 4-city subproblems, each of which can be solved with a 16-qubit budget. Intra-cluster solutions are optimized with QAOA, and are classically stitched to construct the global tour. Our results clearly validate the effectiveness of the hierarchical design of the algorithm, highlighting its feasibility for mid-scale quantum optimizations.**

***Index Terms*—Traveling Salesman Problem (TSP), Quantum Approximate Optimization Algorithm (QAOA), NISQ computing, Clustering-based optimization.**

## I. Introduction

The Traveling Salesman Problem (TSP) is a well-known NP-hard problem that can be applied to find optimal routes in logistics [1]. But, as the number of spatial spots increases, the solution space is subjected to a factorial growth, making the problem inefficient in terms of computing cost. Quantum algorithms offer promising alternatives. Grover's search algorithm (GSA) provides a quadratic speed-up over its classical counterparts for unstructured searches [2], and the Quantum Approximate Optimization Algorithm (QAOA) opens the possibility to determine the optimal path on Noise-Intermediate-Scale Quantum (NISQ) devices [3]. But, their scalability is generally limited by the number of qubits (GSA) or the convergence issue (QAOA) that variational quantum algorithms typically suffers from in NISQ environments.

Researchers have put substantial efforts to solve the TSP problems with quantum algorithms. Sato $et\ al.$proposed a two-step GSA with encoding based on the high-order binary optimization for up to 5-city TSPs [4]. However, their method does not scale to larger or asymmetric graphs. Padmasola $et\ al.$ applied QAOA across multiple hardware types and demonstrated up to 18-city results with photonic Ising machines [5], but their circuit-based solution was limited to the case of 4 cities. Finally Ramezani et al. proposed a theoretical QAOA strategy that reduces the computational complexity from O($n^2$) to O($nlogn$) using graph partitioning [6]. But no performance verification is presented based on circuit-based hardware or simulations.

In this work, we employ a clustering-based QAOA approach and carefully examine its performance to solve TSP problems with circuit-based quantum resources. Optimizing the solutions with QAOA and a classical stitching, we demonstrate that the algorithm successfully extends the scope of computation up to 16-city TSPs.

## II. Methodology

GSA [7] is employed to test the performance of small TSP problems including 4 cities, while a clustering-based QAOA approach is used to scale the size of TSPs up to 16 cities. All the simulations are performed with Qiskit [8] and PennyLane simulator [9] in a single personal computer that is equipped with AMD Ryzen Granite CPU (8 cores and 4.7GHz), 24GB DDR5 memory, and Nvidia Geforce RTX 5070 GPU.

GSA searches for valid tours that produces a total cost lower than a user-defined threshold, requiring $nlogn$ qubits to encode $n$ cities. Including resources needed for distance accumulation, comparison logic and ancilla control, GSA requires a total of 24 qubits to solve a 4-city TSP. For 5-city problems, it needs 56 qubits which cannot be simulated in a classical computer having 24GB memory.

To scale the problem beyond GSA's limitation, the 16-city TSP is manually partitioned into four spatial clusters, where the four 4-city subproblem is solved with QAOA in a 16-qubit budget. Cities are randomly placed, and local distance matrices guide intra-cluster optimizations. The QAOA circuits optimize expected cost while enforcing constraints on valid tours. After solving each cluster, a classical routine stitches the sub-tours into a final global solution. This hybrid design supports modularity and near-term implementation on gate-based quantum hardware in the NISQ regime.

## III. Results and Discussion

### A. GSA: 4-City TSP

GSA is applied to a 4-city TSP with 24 qubits. As shown in Figure 1, dominant bit-strings such as 01101100 (with a probability of 0.04) and 10110001 (with a probability of 0.038) correspond to valid & low-cost tours. The decoded best tour is visualized in Figure 2.

### B. QAOA-Based Global Tour Construction

The 16-city TSP was divided into four 4-city clusters, where each is optimized using QAOA. For each cluster, the shortest valid tour was selected, as shown in Table I. The final global tour, formed by classically stitching the four sub-solutions, is illustrated in Fig. 3.

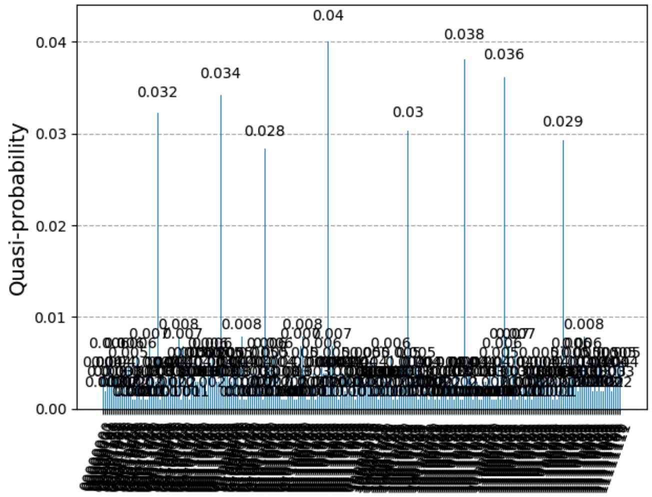

Fig. 1: Quasi-probability distribution obtained by solving a 4-city TSP with GSA. The two dominant cases (01101100 with a probability of 0.4, 10110001 with a probability of 0.038) can serve as the optimal path.

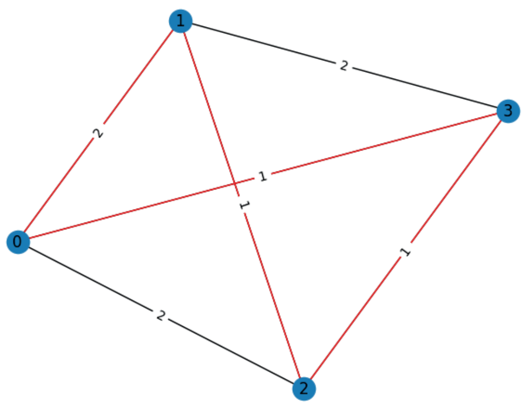

Fig. 2: The optimal tour determined by GSA (solid red lines). Note that each city is encoded with two binary numbers ({00, 01, 10, 11} = city-{0, 1, 2, 3}), so the decoding process (see the caption of Figure 1) is straightforward.

TABLE I: QAOA Outputs for 4-City Clusters

| Cluster | Valid Tours | Shortest Distance |
|---|---|---|
| 0 | 17 | 74.15 |
| 1 | 14 | 74.14 |
| 2 | 4 | 71.52 |
| 3 | 10 | 42.98 |

## IV. Conclusion

This work presents a hybrid quantum-classical approach that can solve TSPs efficiently with NISQ quantum resources. To overcome both the limitation of computing cost and the convergence issue that GSA and full-QAOA have, respectively, we solve large-scale TSPs by decomposing the entire map into small clusters, where each cluster is optimized by QAOA

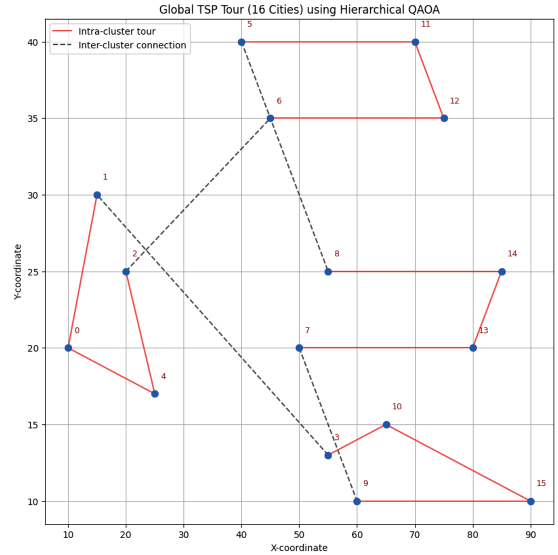

Fig. 3: The final 16-city tour reconstructed with the results of clustering-based QAOA. The solid red lines and the black dashed lines the intra-cluster solutions and the inter-cluster solutions, respectively.

and their sub-solutions are stitched classically. The circuit-based simulations of our hybrid model demonstrates quite nice computing efficiency, securing the solution of up to 16-city TSP problems with 16 qubits. Future directions of our research will include the adaptive clustering and the automated parameter tuning to further enhance the performance.

## Acknowledgment

This work has been carried out under the financial support from the Korea Institute of Science and Technology Information (grant no: P25011).

## References

[1] B. Korte and J. Vygen, *Combinatorial optimization: theory and algorithms*. Springer, 2008.

[2] L. K. Grover, "A fast quantum mechanical algorithm for database search," in *Proceedings of the twenty-eighth annual ACM symposium on Theory of computing*, 1996, pp. 212–219.

[3] E. Farhi, J. Goldstone, and S. Gutmann, "A quantum approximate optimization algorithm," *arXiv preprint arXiv:1411.4028*, 2014.

[4] R. Sato, C. Gordon, K. Saito, H. Kawashima, T. Nikuni, and S. Watabe, "Two-Step Quantum Search Algorithm for Solving Traveling Salesman Problems," *IEEE Transactions on Quantum Engineering*, 2025.

[5] V. Padmasola, Z. Li, R. Chatterjee, and W. Dyk, "Solving the Traveling Salesman Problem via Different Quantum Computing Architectures," *arXiv preprint arXiv:2502.17725*, 2025.

[6] M. Ramezani, S. Salami, M. Shokhmkar, M. Moradi, and A. Bahrampour, "Reducing the Number of Qubits from $n^2$ to $n \log_2(n)$ to Solve the Traveling Salesman Problem with Quantum Computers," *arXiv preprint arXiv:2402.18530*, 2024.

[7] J. Malhotra, O. Chavan, and P. Thakur, "Grover Algorithm: Enhancing the Search Mechanism," in *2025 4th OPJU International Technology Conference (OTCON) on Smart Computing for Innovation and Advancement in Industry 5.0*. IEEE, 2025, pp. 1–8.

[8] IBM Quantum, "Qiskit SDK." [Online]. Available: https://www.ibm.com/quantum/qiskit

[9] Xanadu Quantum Technologies, "Pennylane SDK." [Online]. Available: https://pennylane.ai/