Error Correction Scheme Using Single-Parity-Check Codes and Correlation Receiver Output Levels in Optical SIK DS-CDMA Communications

Akio Tsuneda and Takumi Yoshikawa
Graduate School of Science and Technology, Kumamoto University
Kumamoto, Japan
E-mail: tsuneda@cs.kumamoto-u.ac.jp

***Abstract*—CDMA (code division multiple access) is still an attractive multiple access technique and it has been studied by many researchers, especially most of the recent works are for optical (or visible light) communications. In a correlation receiver of CDMA communication systems based on direct-sequence (DS) spread spectrum techniques, the higher output level is more reliable than the lower one, but the information of the output level is not used in normal decoding and only its sign ($+$ or $-$) is utilized for recovering the information bit. In this paper, a simple error-correction method using single-parity-check (SPC) codes and correlation receiver output levels in optical SIK (sequence inversion keyed) DS-CDMA communications is proposed. Though the SPC codes are 1-bit error-detecting codes (*i.e.*, they have no error-correction ability), we show by computer simulations that the proposed error-correction method can correct some error bits and finally reduce the bit error rate. A theoretical analysis of bit error probability after the proposed error correction is also given.**

***Index Terms*—Optical CDMA, SIK, single-parity-check code, correlation receiver output level, error correction**

I. Introduction

CDMA (code division multiple access) was one of important technologies in 3G (third generation) mobile communication systems [1]. The CDMA techniques still have been studied by various researchers, especially for optical or visible light communications [2]–[5]. Spreading codes (or sequences) play an important role in direct-sequence CDMA (DS-CDMA) communication systems [6]–[8]. In CDMA communications with radio waves, binary spreading sequences are bipolar (± 1) codes. On the other hand, in optical CDMA communications, spreading sequences are unipolar (1 or 0) codes. In the case of unipolar codes, their cross-correlations depend on the number of *collisions* of 1's between unipolar codes [9]. Thus, the design concept of unipolar spreading codes for DS-CDMA is different from that of bipolar codes, and OOC (optical orthogonal code) codes are well known [9]. As a different approach for optical CDMA, a *sequence inversion keyed (SIK)* system using unipolar codes was proposed [10], [11], where the conventional bipolar spreading codes can be used under some conditions.

It is known that spreading codes with some negative auto-correlations can achieve lower bit error rate (BER) in asynchronous DS-CDMA communications than the classical (uncorrelated) spreading codes such as Gold sequences generated by LFSRs (linear feedback shift registers) [12], [13]. Such spreading codes with negative auto-correlation can be designed based on chaos theory for some one-dimensional nonlinear maps [12]–[14]. We designed binary sequences with negative auto-correlation based on the well-known Bernoulli and tent maps [15]. Since the Bernoulli map with finite bits can be realized by linear/nonlinear feedback shift registers (LFSRs/NFSRs), we proposed LFSR/NFSR-based generators of binary spreading codes with negative auto-correlation and revealed that the proposed spreading codes can also reduce BER in asynchronous DS-CDMA systems compared with the Gold sequences [15], [16].

In standard DS-CDMA communications, a receiver judges each information bit based on the correlation receiver (correlator) output. Here, the judgment is only based on the sign (positive or negative) of the correlator output, that is, the output level is not used. However, we consider the level of the correlation receiver output includes some meaningful information for the judgment. In [17], we proposed error-correction methods using the correlation receiver output levels and Hamming codes, and revealed that BER can be reduced by the proposed methods.

In this paper, as a simpler error-control code, we employ single-parity-check (SPC) codes which are 1-bit error-detecting codes, that is, they have no error-correction ability. We propose a simple error-correction method using SPC codes and correlation receiver output levels in DS-CDMA communications, where the *Wagner decoding rule* [18], [19] is applied to multiple-access interference (MAI) environments of DS-CDMA. By computer simulations, the BER characteristics of asynchronous SIK DS-CDMA communications using the proposed error-correction method is investigated. Finally, a theoretical analysis of bit error probability after the proposed error correction is given under some assumptions. Please note that the main purpose of this paper is not to achieve lower bit error probabilities than the conventional error-control codes, but to confirm that the *Wagner decoding rule* is also effective for MAI environments of DS-CDMA by simulations and theoretical analysis.

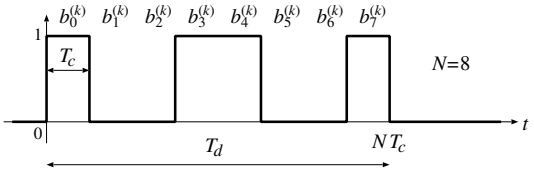

Fig. 1. An example of spreading signal $c^{(k)}(t)$ of Eq.(1) ($N = 8$).

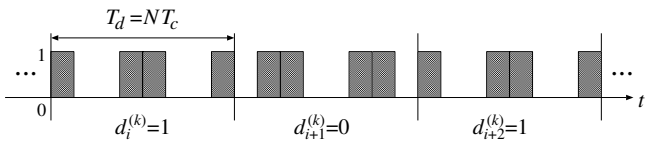

Fig. 2. An example of SIK signal $s^{(k)}(t)$ defined by Eq.(3) ($N = 8$).

II. SIK DS-CDMA Communications

In this paper, we employ SIK DS-CDMA scheme which is one of optical CDMA communications [10], [11]. Let $\{b_j^{(k)}\}_{j=0}^{N-1}$ be the k-th user's unipolar spreading code of even period N and assume it is balanced, that is, the number of 1's is equal to $N/2$. Define the spreading signal by

$$c^{(k)}(t) = \sum_{j=0}^{N-1} b_j^{(k)} P_{T_c}(t - jT_c), \quad b_j^{(k)} \in \{0, 1\}, \qquad (1)$$

where $P_T(t)$ is a unit rectangular pulse of duration T defined by

$$P_T(t) = \begin{cases} 1 & (0 \le t \le T), \\ 0 & (\text{otherwise}), \end{cases} \qquad (2)$$

and T_c is chip duration. An example of spreading signal $c^{(k)}(t)$ defined by Eq.(1) is shown in Fig.1.

In SIK DS-CDMA communications, the k-th user's transmitted signal $s^{(k)}(t)$ is written as

$$s^{(k)}(t) = \sum_{i=-\infty}^{\infty} \Big\{ d_i^{(k)} c^{(k)}(t - iT_d) + (1 - d_i^{(k)})(1 - c^{(k)}(t - iT_d)) \Big\}, \qquad (3)$$

where $d_i^{(k)}$ ($\in \{0, 1\}$) is the k-th user's unipolar data sequence and T_d is data duration given by $T_d = NT_c$. That is, if $d_i^{(k)} = 1$, the spreading signal $c^{(k)}(t - iT_d)$ is transmitted, and if $d_i^{(k)} = 0$, the spreading signal $1 - c^{(k)}(t - iT_d)$ (*bit-flipped* waveform) is transmitted. An example of SIK signal $s^{(k)}(t)$ defined by Eq.(3) is shown in Fig.2.

In asynchronous SIK DS-CDMA communications with K users, the received optical signal $r(t)$ is given by

$$r(t) = \sum_{k=1}^{K} s^{(k)}(t - \tau_k) + w(t), \qquad (4)$$

where the amplitude of each transmitted signal is normalized to 1, τ_k is the relative time delay ($0 \le \tau_k < T$), and $w(t)$ is channel noise. At the k_1-th receiver, the output of the correlation receiver matched to $s^{(k_1)}(t)$ is given by

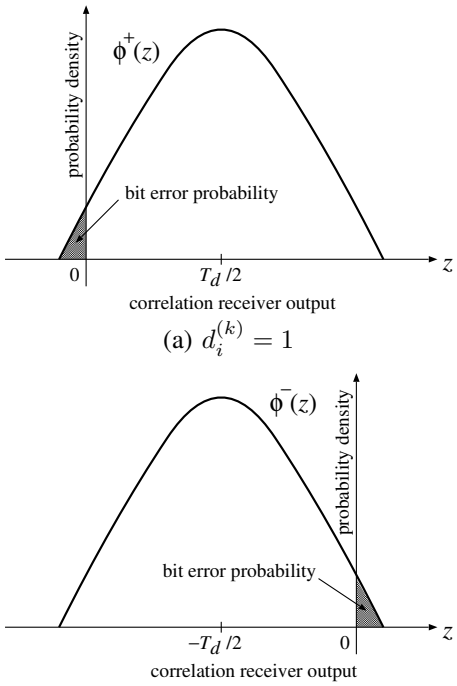

Fig. 3. Overview of distributions of correlation receiver outputs $Z_i^{(k)}$ in SIK DS-CDMA communications.

$$Z_i^{(k_1)} = \int_{iT_d}^{(i+1)T_d} r(t)(2c^{(k_1)}(t) - 1)dt, \qquad (5)$$

$$= (2d_i^{(k_1)} - 1)\frac{T_d}{2} + \underbrace{\int_{iT_d}^{(i+1)T_d} \left\{ \sum_{\substack{k=1 \\ k \ne k_1}}^{K} s^{(k)}(t - \tau_k) \right\} (2c^{(k_1)}(t) - 1)dt}_{\text{MAI}} + \int_{iT_d}^{(i+1)T_d} w(t)(2c^{(k_1)}(t) - 1)dt, \qquad (6)$$

where the second component of the righthand side in Eq.(6) is multiple-access interference (MAI) caused by the cross-correlations between the k_1-th spreading code and the others. The decision at the k_1-th receiver is based on

$$\hat{d}_i^{(k_1)} = \begin{cases} 1 & \text{if } Z_i^{(k_1)} > 0 \\ 0 & \text{if } Z_i^{(k_1)} < 0 \end{cases}, \qquad (7)$$

where $\hat{d}_i^{(k_1)}$ is an estimated value of $d_i^{(k_1)}$. Under the assumption that all spreading sequences $\{b_j^{(k)}\}_{j=0}^{N-1}$ ($k = 1, 2, \cdots, K$) are balanced, this SIK DS-CDMA communications are basically equivalent to the conventional bipolar CDMA [10], [11], [20].

Figure 3 shows overview of distributions (probability density functions) of the correlation receiver outputs $Z_i^{(k)}$, where the area corresponding to the bit error probability is indicated.

III. Proposed Error Correction with SPC Codes and Correlation Receiver Output Levels

A. Single-Parity-Check (SPC) Codes

First, we introduce the well-known single-parity-check (SPC) codes which are one of simplest linear codes [21], [22] used in this paper. Let $\boldsymbol{x} = (x_0, x_1, \cdots, x_{n-2})$ ($x_i \in \{0, 1\}$, $n \geq 2$) be an $(n-1)$-bit information vector. A redundant bit x_{n-1} is determined based on the linear equation

$$x_0 + x_1 + \cdots + x_{n-2} + x_{n-1} = 0, \tag{8}$$

where "+" denotes modulo-2 addition (or exclusive-OR). We can rewrite the above equation as

$$x_{n-1} = x_0 + x_1 + \cdots + x_{n-2}, \tag{9}$$

which implies that x_{n-1} is 0 (or 1) if the number of 1's in $\boldsymbol{x}$ is even (or odd). An n-bit codeword to be transmitted is expressed by $\boldsymbol{u} = (x_0, x_1, \cdots, x_{n-1})$, which is called *single-parity-check (SPC) codes* and denoted by $(n, n-1)$-codes. If one-bit (or odd number of bits) error occurs, we have $x_0 + x_1 + \cdots + x_{n-2} + x_{n-1} = 1$, that is, the receiver can know the received word includes an error bit by checking whether Eq.(8) is satisfied or not (checking $x_{n-1} = 0$ or 1 in Eq.(9)). Thus, the SPC codes can detect 1-bit error in each received word and they are called *1-bit error-detecting codes*.

B. Proposed Error Correction

Noting that the output levels for error bits are relatively small as in Fig.3, we considered that higher correlation receiver outputs are more reliable than lower ones. This coincides with the concept of the *Wagner decoding rule* [18], [19]. Based on this idea, we proposed error-correction methods using the correlation receiver output levels and Hamming codes, and revealed that the bit error rate (BER) can be reduced by the proposed methods in the conventional bipolar DS-CDMA communications [17]. In this paper, we propose a simpler error-correction method using SPC codes and the correlation receiver output levels as follows.

Assume the data sequences are coded by $(n, n-1)$-SPC codes and the k-th receiver recovers n bits per block denoted by $(\hat{x}_0, \hat{x}_1, \cdots, \hat{x}_{n-1})$. The decoding process is done block by block based on the following algorithm.

[Proposed Error-Correction Algorithm]

Step-1: The n output values $Z_0^{(k)}$, $Z_1^{(k)}$, $\cdots$, $Z_{n-1}^{(k)}$ of the k-th correlation receiver for recovering the n bits $(\hat{x}_0, \hat{x}_1, \cdots, \hat{x}_{n-1})$ are stored in memory.

Step-2: Check the parity-check equation by calculating

$$P = \hat{x}_0 + \hat{x}_1 + \cdots + \hat{x}_{n-1}.$$

If $P = 0$, proceed to next block without any correction. If $P = 1$, go to Step-3.

TABLE I
The number of transmitted bits per user in the simulations

$(n, n-1)$	no. of bits/set M	no. of sets S	total no. of bits
$(2, 1)$	1,000	1,000	1,000,000
$(3, 2)$	999	1,001	999,999
$(4, 3)$	1,000	1,000	1,000,000
$(5, 4)$	1,000	1,000	1,000,000
$(6, 5)$	996	1,004	999,984
$(7, 6)$	994	1,006	999,964
$(8, 7)$	1,000	1,000	1,000,000
$(9, 8)$	999	1,001	999,999
$(10, 9)$	1,000	1,000	1,000,000

Step-3: Set $m = \underset{m \in \{0,1,\cdots,n-1\}}{\operatorname{argmin}} |Z_m^{(k)}|$. Then, the corresponding bit $\hat{x}_m$ is corrected ($0 \to 1$ or $1 \to 0$) and proceed to next block.

IV. Asynchronous SIK DS-CDMA Simulation

We perform computer simulations of asynchronous SIK DS-CDMA communications using the proposed error-correction method. In the simulations, we use spreading codes with negative auto-correlation which are generated by nonlinear feedback shift registers (NFSRs) and satisfy the balanced condition (the number of 1's is $N/2$) for SIK DS-CDMA. The details of the NFSR-based spreading codes are described in Appendix A. In order to concentrate our attention on bit errors caused by MAI, we assume $w(t) = 0$ in the simulations. Other simulation conditions are as follows.

- Spreading code length: $N = 64$
- Number of transmitted bits: M bits / user / set
- Number of active users: $K = 20$, 30, 40
- The simulations were performed for S sets of different information bits and random delays.

Here M and S are shown in Table I, where M and S are almost equal to 1,000 and the total number of transmitted bits per user is almost equal to 1,000,000. Note that M should be divided by n (block length).

Tables II–IV show the BER characteristics for $K = 20$, 30, 40, where BER before correction, BER after correction, and BER reduction rate for each n are shown. The BER reduction rate is defined by

$$\text{BER reduction rate} = \frac{\text{(BER before correction)} - \text{(BER after correction)}}{\text{(BER before correction)}}. \tag{10}$$

As in Tables II–IV, the BER reduction rate for $(2, 1)$-codes ($n = 2$) is smaller than that for $(3, 2)$-code ($n = 3$). In $(2, 1)$-codes, the coding is done as $1 \to 11$ and $0 \to 00$, that is, it is the same as the coding for 2-bit repetition codes. Thus, due to the strong correlation between the consecutive two bits, the BER reduction rate is smaller than expected. So we apply an interleaving technique to the $(2, 1)$-codes and the results are also shown in Tables II–IV. We can see that the BER reduction rate for the interleaved $(2, 1)$-codes is larger than that for $(3, 2)$-code as expected. Also, Figure 4 shows the graphs

TABLE II
BER CHARACTERISTICS ($K = 20$)

$(n, n-1)$	BER before correction	BER after correction	BER reduction rate
$(2, 1)$	0.009861	0.004048	0.5895
interleaved	0.009810	0.000806	0.9178
$(3, 2)$	0.009785	0.001408	0.8561
$(4, 3)$	0.009867	0.002198	0.7772
$(5, 4)$	0.009902	0.002684	0.7289
$(6, 5)$	0.009755	0.003138	0.6783
$(7, 6)$	0.009822	0.003592	0.6343
$(8, 7)$	0.009993	0.004100	0.5897
$(9, 8)$	0.009963	0.004410	0.5574
$(10, 9)$	0.009948	0.004776	0.5199

TABLE III
BER CHARACTERISTICS ($K = 30$)

$(n, n-1)$	BER before correction	BER after correction	BER reduction rate
$(2, 1)$	0.028220	0.014236	0.4955
interleaved	0.028170	0.004192	0.8512
$(3, 2)$	0.027994	0.007328	0.7382
$(4, 3)$	0.028257	0.010174	0.6399
$(5, 4)$	0.028455	0.012764	0.5514
$(6, 5)$	0.028043	0.014282	0.4907
$(7, 6)$	0.028126	0.015979	0.4319
$(8, 7)$	0.028227	0.017572	0.3775
$(9, 8)$	0.028138	0.018658	0.3369
$(10, 9)$	0.028225	0.019836	0.2972

TABLE IV
BER CHARACTERISTICS ($K = 40$)

$(n, n-1)$	BER before correction	BER after correction	BER reduction rate
$(2, 1)$	0.049532	0.028758	0.4194
interleaved	0.049156	0.010806	0.7802
$(3, 2)$	0.049929	0.019102	0.6174
$(4, 3)$	0.049886	0.024636	0.5062
$(5, 4)$	0.050096	0.029886	0.4034
$(6, 5)$	0.049695	0.033443	0.3270
$(7, 6)$	0.049603	0.036527	0.2636
$(8, 7)$	0.049958	0.039114	0.2171
$(9, 8)$	0.049833	0.040770	0.1819
$(10, 9)$	0.049874	0.042784	0.1422

of the BER characteristics (before/after correction) and Figure 5 shows the graphs of the BER reduction rate versus n, where the results of the interleaved $(2, 1)$-codes are used for $n = 2$.

From these tables and figures, we can find that the BERs are considerably reduced by the proposed error-correction method, though the SPC codes themselves have no error-correction ability. The BER reduction rate decreases as n increases (*i.e.*, as the code rate $(n-1)/n$ decreases).

V. THEORETICAL ANALYSIS

Now we discuss the theoretical bit error probability after the proposed error correction using $(n, n-1)$-SPC codes.

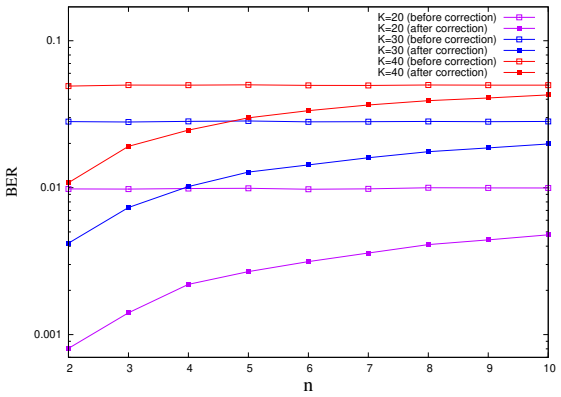

Fig. 4. BER characteristics (BER versus n).

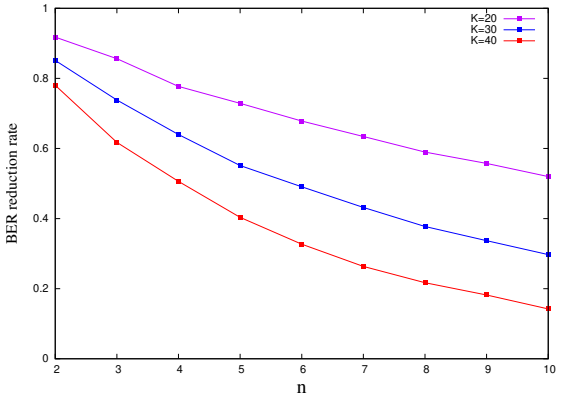

Fig. 5. BER reduction rate versus n.

Let $\phi^+(z)$ and $\phi^-(z)$ be the probability density functions of correlation receiver outputs $Z_i^{(k)}$ shown in Fig.3 (a) and (b), respectively. For simplicity, we assume

$$\phi^+(z) = \phi^-(-z), \tag{11}$$

and the bit error probability p is given by

$$p = \int_{-\infty}^{0} \phi^+(z)dz = \int_{0}^{\infty} \phi^-(z)dz. \tag{12}$$

The conditional probability density function of the output level (absolute value of the correlation receiver output) of the error bits is given by

$$f_e(z) = \begin{cases} \phi^-(z)/p & (z \geq 0) \\ 0 & (z < 0) \end{cases}. \tag{13}$$

Similarly, the conditional probability density function of the output level of the correct (non-error) bits is given by

$$f_c(z) = \begin{cases} \phi^+(z)/(1-p) & (z \geq 0) \\ 0 & (z < 0) \end{cases}. \tag{14}$$

Next, assume a received word of length n includes s error bits ($n-s$ correct bits) and the n output levels are independent of one another (*i.e.*, bit errors are memoryless). If s is an odd

number, one bit, whose output level is minimum among the n output levels including all error/non-error bits, is corrected. The error correction is successful if the minimum output level is any one of the s output levels of the error bits, otherwise the error correction is wrong.

Thus, we consider the probability that the error correction is successful. First, the probability density function of the minimum value of the independent s random variables (output levels of error bits) according to $f_e(z)$ of Eq.(13) is given by [23]

$$g_e(z) = s(1 - F_e(z))^{s-1} f_e(z), \tag{15}$$

where

$$F_e(z) = \int_0^z f_e(z)dz. \tag{16}$$

Similarly, the probability density function of the minimum value of the independent $n-s$ output levels (random variables) according to $f_c(z)$ of Eq.(14) is given by

$$g_c(z) = (n - s)(1 - F_c(z))^{n-s-1} f_c(z), \tag{17}$$

where

$$F_c(z) = \int_0^z f_c(z)dz. \tag{18}$$

Next, let Z_e and Z_c be random variables according to $g_e(z)$ and $g_c(z)$, respectively. Obviously, the error correction is successful when $Z_e < Z_c$. Then, consider a random variable $Z_d = Z_c - Z_e$, where we also assume Z_e and Z_c are independent of each other. The probability density function of Z_d is given by [23]

$$g_d(z) = \int_0^\infty g_c(x) g_e(x - z)dx. \tag{19}$$

Thus, the probability that the 1-bit error correction is successful for a received word of length n with s error bits, denoted by $q_s^{(n)}$, is obtained by

$$q_s^{(n)} = \begin{cases} \int_0^\infty g_d(z)dz & (s < n) \\ 1 & (s = n) \end{cases}, \tag{20}$$

where s is an odd number satisfying $1 \le s \le n$. Note that the 1-bit error correction is always correct when all the n (odd number) bits are error bits, that is, $q_n^{(n)} = 1$ for any odd number $n \ge 3$.

Let L be the total number of transmitted codewords of length n for a user, that is, the total number of transmitted bits is nL per user. Using the bit error probability p, the expected value of the total number of error bits (before error correction) is nLp. In the proposed error-correction algorithm, the 1-bit error correction will be done when s is an odd number in a received word of length n. The probability that an odd number $2r - 1$ $(r = 1, 2, \cdots)$ of error bits occur in a received word of length n is expressed by

$$p_{2r-1}^{(n)} = \binom{n}{2r-1} p^{2r-1}(1 - p)^{n-(2r-1)}, \tag{21}$$

and the expected value of the total number of received words with $2r - 1$ error bits is $Lp_{2r-1}^{(n)}$. Thus, $Lp_{2r-1}^{(n)} q_{2r-1}^{(n)}$ error bits are correctly corrected and the $Lp_{2r-1}^{(n)}(1 - q_{2r-1}^{(n)})$ error bits are incorrectly corrected. After the error correction, the expected value of the number of error bits is decreased by $Lp_{2r-1}^{(n)}(2q_{2r-1}^{(n)} - 1)$, which gives the total number of error bits after error correction is calculated by

$$nLp - L\sum_{r=1}^{m} p_{2r-1}^{(n)}(2q_{2r-1}^{(n)} - 1), \tag{22}$$

where $n = 2m$ $(m = 1, 2, \cdots)$ or $n = 2m-1$ $(m = 2, 3, \cdots)$. Dividing Eq.(22) by nL, we have the theoretical (estimated) bit error probability $\hat{p}$ after error correction, given by

$$\hat{p} = p - \frac{1}{n}\sum_{r=1}^{m} p_{2r-1}^{(n)}(2q_{2r-1}^{(n)} - 1). \tag{23}$$

For example, the expressions of $\hat{p}$ and related parameters and functions for $(2, 1)$ and $(3, 2)$ codes are shown in Table V. To confirm the validity of the theoretical analysis, we compare the simulation results for $(2, 1)$ (interleaved) and $(3, 2)$ codes given in the previous section and the theoretical estimates $\hat{p}$ given in Table V. The comparison is shown in Table VI, where we assume $\phi^+(z)$ and $\phi^-(z)$ are Gaussian distributions whose mean and variance are obtained by the simulations. The integrals for $F_c(z)$, $g_d(z)$, and $q_s^{(n)}$ were calculated by numerical integration. Though the simulation BERs and the theoretical estimates are somewhat different (especially for small K), we see that rough estimates can be obtained by Eq.(23).

TABLE V
EXPRESSIONS OF THEORETICAL $\hat{p}$ AND RELATED PARAMETERS AND FUNCTIONS FOR $(2, 1)$ AND $(3, 2)$ CODES.

	$(2,1)$-code	$(3,2)$-code	
s	1	1	3
$g_e(z)$	$f_e(z)$	$f_e(z)$	–
$g_c(z)$	$f_c(z)$	$2(1 - F_c(z))f_c(z)$	–
$q_s^{(n)}$	$q_1^{(2)}$	$q_1^{(3)}$	1
$p_s^{(n)}$	$2p(1-p)$	$3p(1-p)^2$	p^3
$\hat{p}$	$p - \frac{1}{2}p_1^{(2)}(2q_1^{(2)} - 1)$	$p - \frac{1}{3}p_1^{(3)}(2q_1^{(3)} - 1) - \frac{1}{3}p^3$	

TABLE VI
BER AFTER ERROR CORRECTION OF SIMULATION RESULTS AND THEORETICAL ESTIMATES FOR $(2, 1)$ AND $(3, 2)$ CODES.

$(n, n-1)$	K	simulation	theoretical estimate ($\hat{p}$)
$(2, 1)$	20	0.000806	0.000472
	30	0.004192	0.003425
	40	0.010806	0.009938
$(3, 2)$	20	0.001408	0.000896
	30	0.007328	0.006349
	40	0.019102	0.017730

VI. Conclusion

We have proposed a simple error-correction method for SIK DS-CDMA communications using the correlation receiver output levels and SPC codes, which is based on the *Wagner decoding rule*. Simulation results have shown that the proposed method can reduce BER in spite of the fact that the SPC codes have no error-correction ability, that is, the *Wagner decoding rule* is also effective for SIK DS-CDMA communications. Thus we can conclude that the correlation receiver output levels include some meaningful information for CDMA decoding. It should be noted that the proposed method can be used in the conventional bipolar DS-CDMA communications. Finally, the theoretical analysis of bit error probability after the proposed error correction is discussed and its validity has been confirmed.

Appendix

NFSRs with k stages can generate huge number of maximum-period sequences of period $N = 2^k$ called *de Bruijn sequences* and the number of different de Bruijn sequences is $2^{2^{k-1}-k}$ [24], [25].

Figure 6 shows a generator of spreading codes with negative auto-correlation based on an NFSR generating a de Bruijn sequence, where the combinational logic circuit has 3-input and 1-output. The logic function $f(a_0, a_1, a_2)$ used in this paper is defined by

$$f(a_0, a_1, a_2) = \begin{cases} 1 & a_0a_1a_2 \in \{010, 011, 100, 110\}, \\ 0 & \text{otherwise}, \end{cases}$$

which is designed based on the chaos theory for the Bernoulli map [15], [16]. Such spreading codes can reduce BER in asynchronous DS-CDMA communications (with bipolar codes) compared to the original de Bruijn sequences and Gold sequences [15]. Note that the spreading codes (including the original de Bruijn sequences) are completely balanced, that is, the number of 1's in a period is exactly equal to $N/2$ $(= 2^{k-1})$. This means they are suitable for SIK DS-CDMA communications [20]

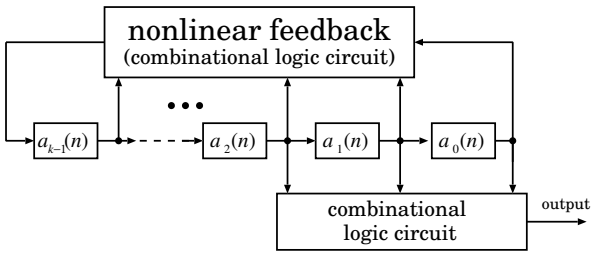

Fig. 6. Generator of spreading codes with negative auto-correlation using a k-stage NFSR and a combinational logic circuit with 3-input/1-output.

References

[1] S. Glisic and B. Vucetic, Spread spectrum CDMA systems for wireless communications, Artech House, 1997.

[2] Y. Qiu, S. Chen, H.-H. Chen, and W. Meng, "Visible light communications based on CDMA technology," IEEE Wireless Communications, vol.25, no.2, pp.178–185, Apr. 2018.

[3] T. K. Matsushima, S. Yamasaki, K. Ono, and H. Tanaka, "Visible-Light CDMA Communications Using Inverted Spread Sequences," Electronics, vol.11, no.12, 1823, Jun. 2022.

[4] D. Chen, Q. Wang, J. Wang, J. Jin, H. Lu, and L. Feng, "Performance evaluation of ZCC and OZCZ code set in an integrated VLCP-CDMA system," IEEE Photonics Technology Letters, vol.34, no.16, pp.846–849, Aug. 2022.

[5] B.-C. Yeh, "Establishing a spectral Optical CDMA system involving the use of two mutually orthogonal states of polarizations using 1-D two-distinct codes with two-code keying," IEEE Access, vol.12, pp.53018–53030, 2024.

[6] M. B. Pursley, "Performance evaluation for phase-coded spread spectrum multiple-access communication—part I: System analysis," IEEE Trans. Commun., vol.COM-25, no.8, pp.795–799, Aug. 1977.

[7] D. V. Sarwate and M. B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences," Proc. IEEE, vol.68, no.3, pp.593–619, May 1980.

[8] P. Fan and M. Darnell, Sequence design for communications applications, Research Studies Press, 1996.

[9] F. R. K. Chung, J. A. Salehi, and V. K. Wei, "Optical orthogonal codes: design, analysis and applications," IEEE Trans. Information Theory, vol.35, no.3, pp.595–604, 1989.

[10] M. J. Parham, C. Smythe, and B. L. Weiss, "Code division multiple-Access techniques for use in optical-fibre local-area networks," Electron. & Commun. Eng. J., pp.203–212, 1992.

[11] T. O'Farrell and S. Lockmann, "Performance analysis of an optical correlator receiver for SIK DS-CDMA communication systems," Electronics Letters, vol.30, no.1, pp.63–65, 1994.

[12] R. Rovatti and G. Mazzini, "Interference in DS-CDMA systems with exponentially vanishing autocorrelations: chaos-based spreading is optimal," Electronics Letters, vol.34, no.20, pp.1911–1913, Oct. 1998.

[13] G. Mazzini, R. Rovatti, and G. Setti, "Interference minimization by auto-correlation shaping in asynchronous DS-CDMA systems: chaos-based spreading is nearly optimal," Electronics Letters, vol.35, no.13, pp.1054–1055, Jun. 1999.

[14] A. Tsuneda, "Design of binary sequences with tunable exponential autocorrelations and run statistics based on one-dimensional chaotic maps," IEEE Trans. Circuits Syst. I, vol.52, no.2, pp.454–462, Feb. 2005.

[15] A. Tsuneda, D. Yoshioka, and T. Hadate, "Design of spreading sequences with negative auto-correlations realizable by nonlinear feedback shift registers," in Proc. of the 8th IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA 2004), Sydney, Australia, Aug./Sep. 2004, pp.330–334.

[16] A. Tsuneda and Y. Miyazaki, "Binary spreading sequences with negative auto-correlation based on chaos theory and Gold sequences for application to asynchronous DS/CDMA communications," IEICE Trans. Fundamentals, vol.E93-A, no.11, pp.2307–2311, Nov. 2010.

[17] Y. Tsuruda and A. Tsuneda, "Study on error-correcting using output level of correlation receivers and Hamming codes in CDMA communications," In Proc. of the 9th International Conference on ICT Convergence (ICTC 2018), Jeju Island, Korea, Oct. 2018, pp.234–236.

[18] R. Silverman and M. Balser, "Coding for constant-data-rate systems," Trans. IRE Prof. Group Inf. Theory, vol.4, no.4, pp.50–63, Sep. 1954.

[19] J. Snyders and Y. Be'ery, "Maximum likelihood soft decoding of binary block codes and decoders for the Golay codes," IEEE Trans. Inform. Theory, vol.35, no.5, pp.963–975, Sep. 1989.

[20] A. Tsuneda and T. Yoshida, "Performance evaluation of asynchronous DS/CDMA communications using unipolar codes," Proc. of 2011 European Conference on Circuit Theory and Design, Linkoping, Sweden, Aug. 2011, pp.669–672.

[21] W. W. Peterson and E. J. Weldon Jr. Error-correcting codes (2nd ed.), The MIT Press, 1972.

[22] B. P. Lathi, Modern digital and analog communication systems (3rd ed.), Oxford Univ. Press, 1998.

[23] F. M. Dekking, C. Kraaikamp, H. P. Lopuhaä, and L. E. Meester, A Modern Introduction to Probability and Statistics, Springer, 2005.

[24] N. G. de Bruijn, "A combinatorial problem," Nederl. Akad. Wetensch. Proc., vol.49, no.7, pp.758–764, 1946.

[25] S. W. Golomb, Shift Register Sequences (revised ed.), Aegean Park Press, 1982.