Environment-aware Direct Position Estimation with LEO Systems

Duhui Yang, Hyunwoo Park, and Sunwoo Kim
Department of Electronic Engineering Hanyang University, Seoul, South Korea
Email: {yangdh95, stark95, remero}@hanyang.ac.kr

***Abstract*—This paper proposes an environment-aware direct position estimation (EDP) method for multi-low earth orbit (LEO) positioning in urban environments. Although LEO constellations provide strong received power, practical positioning is hindered by frequent satellite blockage and severe multipath. Moreover, the limited bandwidth of downlink reference signals reduces delay resolution, making line of sight (LoS)/non LoS (NLoS) separation unreliable and degrading conventional estimators. To address this, we exploit environmental information to predict candidate specular reflection points and reconstruct a geometry-consistent NLoS channel model. We then model dominant NLoS components as structured paths parameterized by the UE position and formulate the augmented objective function with the NLoS channel model. Simulation results show that the proposed method outperforms in terms of positioning root mean squared error (RMSE) compared to the conventional methods.**

***Index Terms*—LEO positioning, direct position estimation, multipath, Doppler**

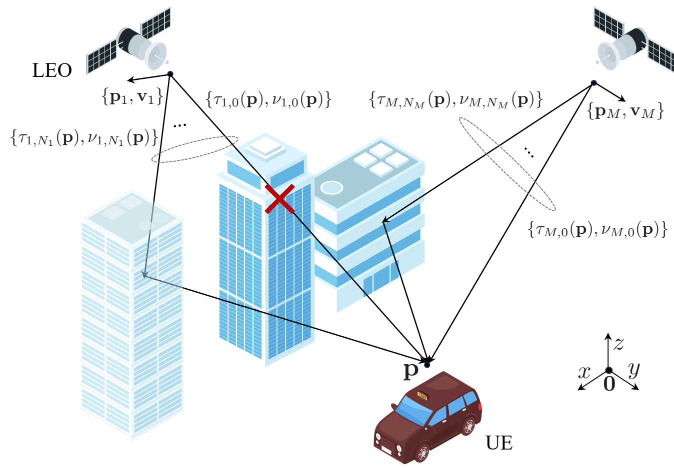

Fig. 1. Positioning scenario with multi-LEO systems in urban environment.

I. Introduction

Low Earth orbit (LEO) constellations are rapidly becoming a key enabler of next-generation positioning, navigation, and timing (PNT) services, complementing or extending conventional GNSS [1]–[4]. In particular, LEO platforms provide (i) stronger received power due to their lower altitude and (ii) pronounced Doppler shifts induced by high orbital velocities, both of which can be exploited to improve positioning robustness and accuracy [1], [5]. This trend is further accelerated by the integration of communication and navigation functionalities in emerging LEO/non terestrial network (NTN) systems, where positioning is expected to operate as a built-in capability using downlink reference signals already transmitted for communication purposes [4], [6], [7].

Despite this promise, accurate LEO-based positioning in dense urban environments remains challenging [1], [2]. First, satellite blockage frequently removes many links, leaving only a small set of usable measurements at any time [2], [3]. Second, navigation-integrated LEO communication systems often employ narrowband reference signals, which directly limit delay resolution [1], [4], [6]. This is critical in urban channels. With limited bandwidth, multipath components become less resolvable in delay, making separation between line of sight (LoS) and strong non LoS (NLoS) paths difficult. As a consequence, conventional estimators that assume a dominant LoS component can suffer severe bias when specular reflections dominate or when LoS is intermittently blocked. While Doppler-based positioning is attractive under narrow bandwidth since Doppler resolution can still be improved via temporal observation, Doppler measurements are also distorted by multipath when multiple paths have comparable energy or when the strongest path is not LoS [1], [5].

Meanwhile, prior information about dominant reflectors around the UE can provide valuable geometric constraints on the channel, enabling consistent hypotheses for strong specular multipath components [8]. Motivated by this, we develop an environment-aware direct positioning method for LEO systems. Specifically, dominant multipath is not treated as unmodeled interference. Instead, it is modeled as structured signal components parameterized by the UE position through a set of specular reflections. Building on the direct positioning (DP) framework [9], [10], we formulate a multipath-aware objective by augmenting the LoS model with specular paths and concentrating out the unknown complex path gains in closed form. We use coarse positioning and DP refinement with the augmented objective function in sequence to improve urban positioning robustness and accuracy, which is the main goal of this paper.

This work was supported by the National Research Foundation of Korea under Grant RS-2024-00409492.

II. System model and problem definition

A. 2D received signal model

We consider an OFDM downlink reference signal transmitted from M single-antenna LEO satellites to a single-antenna UE. The m-th satellite position and velocity are denoted by $\mathbf{p}_m \in \mathbb{R}^{3\times 1}$ and $\mathbf{v}_m \in \mathbb{R}^{3\times 1}$ for $m \in \{1,\ldots,M\}$, and are assumed known at the ground base station (BS). The UE is located at an unknown position $\mathbf{p} \in \mathbb{R}^{3\times 1}$ and is quasi-static during the observation interval.

The reference signal occupies K subcarriers and L OFDM symbols with subcarrier spacing Δ_f. The OFDM symbol duration is $T_{\rm sym} = T + T_{\rm cp}$ with $T = 1/\Delta_f$. The UE estimates the channel for each satellite and reports it to the BS. Let $\mathbf{Y}_m[k,\ell]$ denote the received signal matrix corresponding to subcarrier $k \in \{0,\ldots,K-1\}$ and OFDM symbol $\ell \in \{0,\ldots,L-1\}$. $\mathbf{Y}_m[k,\ell]$ is defined as

$$\mathbf{Y}_m[k,\ell] = \mathbf{H}_m[k,\ell] + \bar{\mathbf{H}}_m[k,\ell] + \mathbf{N}_m[k,\ell] \qquad (1)$$

where $\mathbf{H}_m[k,\ell]$ and $\bar{\mathbf{H}}_m[k,\ell]$ are the LoS and NLoS channel matrix, respectively, and $\mathbf{N}_m[k,\ell] \sim \mathcal{CN}(0,\sigma^2)$ are measurement noise and i. i. d for different entities $\forall m \in \{1,\ldots,M\}$.

The LoS channel matrix is denoted as

$$\mathbf{H}_m(\mathbf{p})[k,\ell] = \alpha_m(\mathbf{p}) e^{-j2\pi k\Delta_f \tau_m(\mathbf{p})} \cdot e^{j2\pi \ell T \nu_m(\mathbf{p})} \qquad (2)$$

where $\alpha_m(\mathbf{p}) = \rho_m e^{j2\pi f_c \tau_m(\mathbf{p})}$, is the complex channel gain with carrier phase, $\rho_m \in \mathbb{C}$ captures attenuation (e.g., path loss, penantration loss), $\tau_m(\mathbf{p}) = \|\mathbf{p}-\mathbf{p}_m\|/c + \delta_m \in \mathbb{R}$ and $\nu_m(\mathbf{p}) = \frac{\mathbf{v}_m^{\mathsf{T}}(\mathbf{p}-\mathbf{p}_m)}{c\|\mathbf{p}-\mathbf{p}_m\|} + \dot{\delta}_m \in \mathbb{R}$ are the propagation delay and Doppler shift of the m-th satellite to UE path. Here, δ_m and $\dot{\delta}_m$ are the propagation delay offset and its rate of change, including the clock offset, drift, and the atmospheric delay (e.g., ionospheric and tropospheric effect)

The NLoS channel matrix consists of N_m specular paths reflected from the adjacent buildings, denoted by its reflecting point as $\mathbf{r}_{m,n}(\mathbf{p}) \in \mathbb{C}^{3\times 1}$ and it's denoted as

$$\bar{\mathbf{H}}_m(\mathbf{p})[k,\ell] = \sum_{n=1}^{N_m} \alpha_{m,n}(\mathbf{p}) \cdot e^{-j2\pi k\Delta_f \tau_{m,n}(\mathbf{p})} \cdot e^{j2\pi \ell T \nu_{m,n}(\mathbf{p})} \qquad (3)$$

where $\alpha_{m,n}(\mathbf{p}) = \rho_{m,n} e^{j2\pi f_c \tau_{m,n}(\mathbf{p})}$ is the complex channel gain of n-th including carrier phase, $\tau_{m,n}(\mathbf{p}) = (\|\mathbf{p}_m - \mathbf{r}_{m,n}(\mathbf{p})\| + \|\mathbf{r}_{m,n}(\mathbf{p}) - \mathbf{p}\|)/c + \delta_m$ is the time delay of the n-th specular path and $\nu_{m,n}(\mathbf{p}) = (\mathbf{v}_m^{\mathsf{T}}(\mathbf{r}_{m,n}(\mathbf{p}) - \mathbf{p}_m))/(c\|\mathbf{r}_{m,n}(\mathbf{p}) - \mathbf{p}_m\|) + \dot{\delta}_m$ is the Doppler shift of th n-th specular path.

B. Problem definition

Our primary goal is to estimate $\mathbf{p}$ from the $\{\mathbf{Y}_m\}_{m=1}^M$, whose entries are parametrized through $\{\nu_m(\mathbf{p}), \tau_m(\mathbf{p})\}_{m=1}^M$ including nuisance parameter $\{\delta_m, \dot{\delta}_m\}_{m=1}^M$. Only considering the LoS paths, our objective is to estimate the position by solving the following optimization problem:

$$\hat{\mathbf{p}} = \arg\min_{\mathbf{p}} \sum_{m=1}^{M} \|\mathbf{y}_m - \alpha_m(\mathbf{p})\mathbf{h}_m(\mathbf{p})\|^2 \qquad (4)$$

where $\mathbf{y}_m = \mathrm{vec}(\mathbf{Y}_m) \in \mathbb{C}^{KL\times 1}$ is the vectorized observation matrix, and $\mathbf{h}_m(\mathbf{p}) = \alpha_m(\mathbf{p})\mathbf{c}(\tau_m(\mathbf{p})) \otimes \mathbf{b}(\tau_m(\mathbf{p})) \in \mathbb{C}^{KL\times 1}$ denotes the LoS channel vector, which is basis function to find $\mathbf{p}$. The delay and Doppler steering vectors are given by $\mathbf{c}(\tau) \in \mathbb{C}^{K\times 1}$ and $\mathbf{b}(\nu) \in \mathbb{C}^{L\times 1}$, whose elements are defined as $[\mathbf{c}(\tau)]_k = e^{-j2\pi k\Delta f\,\tau}$ and $[\mathbf{b}(\nu)]_\ell = e^{j2\pi \ell T\,\nu}$, respectively.

Here, in urban environments, NLoS components $\{\bar{\mathbf{H}}_m(\mathbf{p})\}_{m=1}^M$ act as significant interference to the direct path, particularly under bandwidth-limited scenarios where the delay resolution is insufficient to resolve multipath components. Consequently, the solution of (4) deviates from the true maximum likelihood (ML) estimate, leading to a notable degradation in positioning accuracy.

III. Environment-aware DP

The proposed method exploits reflector information to reconstruct dominant specular paths and mitigate the multipath-induced performance loss of the LoS-only DP in (4). The key idea is to augment the LoS basis $\mathbf{h}_m(\mathbf{p})$ with a set of specular NLoS components predicted by the geometric relationship between satellite, reflector given position, and to directly estimate $\mathbf{p}$ by fitting an augmented basis function to the received observations. The method operates in two stages:

1) Coarse positioning step that obtains an initial estimate $\tilde{\mathbf{p}}$ from delay–Doppler estimation performed independently on each satellite path
2) Refinement step that solves an environment-aware direct positioning problem with the augmented basis initialized at $\tilde{\mathbf{p}}$.

The overall two-stage procedure with basis function augmentation are detailed in the following subsections.

A. Coarse positioning

From (1), the Doppler shift and delay of m-th satellite are estimated via 2D-discrete Fourier transform (DFT) or separate 1D-DFT with non-coherent integration in subcarrier and symbol domain. With the delay and Doppler steering vectors, the objective function of parameter estimations is defined as

$$\{\hat{\tau}_m, \hat{\nu}_m\} = \arg\max_{\tau_m,\nu_m} \left|\mathbf{c}^{\mathsf{H}}(\tau_m)\mathbf{Y}_m\mathbf{b}^*(\nu_m)\right| \qquad (5)$$

where $\hat{\tau}_m$ and $\hat{\nu}_m$ are delay and Doppler estimate of the m-th satellite. With the channel parameter estimates of all satellites $\{\hat{\tau}_m, \hat{\nu}_m\}_{m=1}^M$, the coarse position estimate $\tilde{\mathbf{p}} \in \mathbb{C}^{3\times 1}$ is obtained by solving

$$\tilde{\mathbf{p}} = \arg\min_{\mathbf{p}} \sum_{m=1}^{M} \|\hat{\mathbf{z}} - \mathbf{z}(\mathbf{p})\|^2 \qquad (6)$$

where $\hat{\mathbf{z}} = [\hat{\tau}_1,\ldots,\hat{\tau}_M,\hat{\nu}_1,\ldots,\hat{\nu}_M]^{\mathsf{T}} \in \mathbb{C}^{2M\times 1}$ and $\mathbf{z} = [\tau_1(\mathbf{p}),\ldots,\tau_M(\mathbf{p}),\nu_1(\mathbf{p}),\ldots,\nu_M(\mathbf{p})]^{\mathsf{T}} \in \mathbb{C}^{2M\times 1}$ are the vector of channel parameter estimates and corresponding models. Here, (6) is the nonlinear least squares (NLS) problem solved by a well-known optimization algorithm.

B. Direct positioning with augmented basis

Given the coarse estimate $\tilde{\mathbf{p}}$, the final position estimate is obtained by directly fitting the NLoS path model, including the channel model, to the observed data.

When the reflector information is known with satellite position, the set of reflection points estimates $\{\hat{\mathbf{r}}_{m,n}(\mathbf{p})\}_{n=1}^{N_m}$ is obtained given $\mathbf{p}$. With the reflection point sets, we can reconstruct the NLoS channel model with corresponding specular reflection $n \in \{1, \ldots, N_m\}$. The NLoS channel vector is denoted as

$$\hat{\mathbf{h}}_{m,n}(\mathbf{p}) \triangleq \hat{\mathbf{c}}_{m,n}(\mathbf{p}) \otimes \hat{\mathbf{b}}_{m,n}(\mathbf{p}) \in \mathbb{C}^{KL\times 1} \tag{7}$$

where $\hat{\mathbf{c}}_{m,n}(\mathbf{p})$ and $\hat{\mathbf{b}}_{m,n}(\mathbf{p})$ are constructed from the predicted reflection point $\hat{\mathbf{r}}_{m,n}(\mathbf{p})$ via the corresponding delay and Doppler. Stacking these NLoS channel vector with the LoS channel vector, the basis function is augmented as

$$\hat{\mathbf{A}}_m(\mathbf{p}) = \left[\mathbf{h}_m(\mathbf{p}), \hat{\mathbf{h}}_{m,1}(\mathbf{p}), \ldots, \hat{\mathbf{h}}_{m,N_m}(\mathbf{p})\right] \in \mathbb{C}^{KL\times(N_m+1)} \tag{8}$$

and the corresponding complex path-gain vector

$$\boldsymbol{\alpha}_m \triangleq \left[\alpha_m,\ \alpha_{m,1}, \ldots, \alpha_{m,N_m}\right]^{\mathsf{T}} \in \mathbb{C}^{(N_m+1)\times 1}. \tag{9}$$

With this notation, the multipath-aware signal model becomes

$$\mathbf{y}_m = \hat{\mathbf{A}}_m(\mathbf{p})\boldsymbol{\alpha}_m + \mathbf{n}_m \tag{10}$$

where $\mathbf{n}_m \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_{KL})$.

Finally, the log-likelihood with augmented basis is defined as

$$\mathcal{L}(\mathbf{p}) = \sum_{m=1}^{M} \left\|\mathbf{y}_m - \hat{\mathbf{A}}_m(\mathbf{p})\boldsymbol{\alpha}_m(\mathbf{p})\right\|^2. \tag{11}$$

Here, the complex gain vector is expressed as a closed form with $\hat{\mathbf{A}}(\mathbf{p})$

$$\boldsymbol{\alpha}_m(\mathbf{p}) = \left(\hat{\mathbf{A}}_m^{\mathsf{H}}(\mathbf{p})\hat{\mathbf{A}}_m(\mathbf{p})\right)^{-1}\hat{\mathbf{A}}_m^{\mathsf{H}}(\mathbf{p})\mathbf{y}_m \tag{12}$$

Substituting (12) into (11) yields

$$\mathcal{L}(\mathbf{p}) = \sum_{m=1}^{M} \left\|\mathbf{P}^{\perp}_{\hat{\mathbf{A}}_m(\mathbf{p})}\mathbf{y}_m\right\|^2 \tag{13}$$

where $\mathbf{P}^{\perp}_{\hat{\mathbf{A}}_m(\mathbf{p})} \triangleq \mathbf{I}_{KL} - \hat{\mathbf{A}}_m(\mathbf{p})\left(\hat{\mathbf{A}}_m^{\mathsf{H}}(\mathbf{p})\hat{\mathbf{A}}_m(\mathbf{p})\right)^{-1}\hat{\mathbf{A}}_m^{\mathsf{H}}(\mathbf{p})$ is the projection matrix onto the subspace spanned by $\hat{\mathbf{A}}_m(\mathbf{p})$.

The proposed method then estimates the UE position as

$$\hat{\mathbf{p}} = \arg\min_{\mathbf{p}\in\mathcal{P}} \mathcal{L}(\mathbf{p}) \tag{14}$$

where the search region $\mathcal{P}$ is restricted to a small 3D neighborhood around the coarse estimate, e.g., $\mathcal{P} = \left\{\mathbf{p} : \|\mathbf{p}-\tilde{\mathbf{p}}\| \leq R\right\}$. In practice, (14) is solved via a grid search over $\mathcal{P}$: for each candidate point, the reflector information is used to obtain $\{\hat{\mathbf{r}}_{m,n}(\mathbf{p})\}$, the corresponding delay/Doppler steering vectors are updated to form $\mathbf{A}_m(\mathbf{p})$, and $\mathcal{L}(\mathbf{p})$ in (13) is evaluated. The grid point that minimizes $\mathcal{L}(\mathbf{p})$ is taken as the fine position estimate.

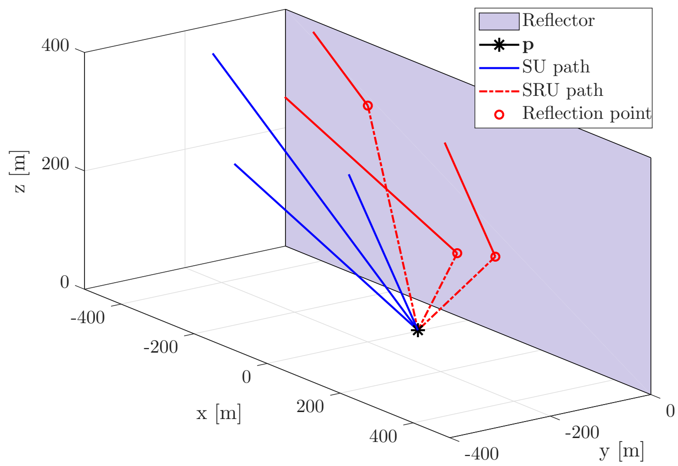

Fig. 2. LoS and NLoS path of the simulation environments. Here, SU and SRU path stand for satellite-UE and satellite-reflection-UE path, respectively.

IV. Simulation Results

A. Simulation setup

For simulation, the satellite geometry is modeled in a local Cartesian coordinate system centered at the $[0, 0, 0]^{\mathsf{T}}$. We consider $M = 3$ LEO satellites at an altitude of 600 km and elevation angle of 45°. Their azimuth angles are set as $\{200°,\ 253.3°,\ 306.7°\}$, and the corresponding orbital inclinations are chosen such that these azimuth/elevation angles are maintained over the observation interval. The UE position is fixed at $\mathbf{p} = [0,\ -100,\ 1.5]^{\mathsf{T}}$ m. the carrier frequency is set to $f_c = 2$ GHz with a subcarrier spacing of $\Delta_f = 60$ kHz. The OFDM related parameters are set as $K = 128$ subcarriers and$L = 8$ OFDM symbols. The average received SNR at the UE is fixed to 10 dB for all links, which is representative of typical LEO environments. The urban environment is represented by a single vertical reflecting wall. As shown in Fig. 2, the wall lies on the plane $y = 0$ and is parallel to the x-axis, with width $w = 1{,}000$ m along the x-direction and height $h = 400$ m along the z-direction. For each satellite, we assume that the LoS path is always available and that exactly one first-order specular reflection is present on this wall, i.e., $N_m = 1$ for all $m \in \{1, \ldots, M\}$. Consequently, the channel for each satellite consists of one LoS path and one specular path, both parameterized by the UE position. Here, the redundant offset and the UE height are assumed as known values.

B. Simulation results

To quantify the overall performance in a compact form, Table I shows the 2D positioning root mean squared error (RMSE) of the conventional two-step method, LoS-only DP, and the proposed EDP. Here, for the 2D-grid search is used to find the solution of DP objective function with $R = 10$ m and 0.1 m grid size. The two-step baseline performs worst, with an RMSE of ≈ 10.5 m, mainly due to multipath-induced bias: since the peak-based delay/Doppler extraction does not model the specular component, the dominant NLoS path contaminates the estimates and causes a systematic position shift.

TABLE I
POSITIONING ERROR AT $\mathrm{SNR} = 10\,\mathrm{dB}$.

Method	2D positioning RMSE [m]
Two-step	10.5
LoS-only DP	5.7
EDP (proposed)	3.3

LoS-only DP improves upon the two-step baseline, reducing the RMSE to $\approx 5.7\,\mathrm{m}$. In contrast, EDP outperforms the other method with the positioning RMSE of $\approx 3.3\,\mathrm{m}$. This gain stems from explicitly incorporating the predicted specular path ($N_m = 1$ for all m) into the augmented basis function, thereby exploiting the geometric diversity brought by the specular paths.

V. CONCLUSION

We propose an environment-aware direct positioning method for LEO systems exploiting the prior reflector information. By incorporating predicted specular reflection points into the DP formulation and eliminating the unknown complex path gains in closed form, the proposed EDP improves positioning accuracy compared to the two-step baseline and the LoS-only DP. Future work will extend the current specular-only channel model toward more realistic urban propagation conditions, including diffuse scattering, multiple-bounce paths, and reflector uncertainty.

REFERENCES

[1] F. S. Prol *et al.*, "Position, navigation, and timing (PNT) through low earth orbit (LEO) satellites: A survey on current status, challenges, and opportunities," *IEEE Access*, vol. 10, pp. 83 971–84 002, 2022.

[2] H. K. Dureppagari, C. Saha, H. S. Dhillon, and R. M. Buehrer, "NTN-based 6G localization: Vision, role of LEOs, and open problems," *IEEE Wireless Commun.*, vol. 30, no. 6, pp. 44–51, 2023.

[3] H. Sallouha *et al.*, "On the ground and in the sky: A tutorial on radio localization in ground-air-space networks," *IEEE Commun. Surveys Tuts.*, vol. 27, no. 1, pp. 218–258, 2025.

[4] S. Saleh *et al.*, "Integrated 6G TN and NTN localization: Challenges, opportunities, and advancements," *IEEE Commun. Standards Mag.*, vol. 9, no. 2, pp. 63–71, 2025.

[5] B. McLemore and M. L. Psiaki, "Navigation using Doppler shift from LEO constellations and ins data," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 58, no. 5, pp. 4295–4314, 2022.

[6] J. Ma, P. Zheng, X. Liu *et al.*, "Integrated positioning and communication via LEO satellites: Opportunities and challenges," *arXiv preprint arXiv:2411.14360*, 2024.

[7] *Study on New Radio (NR) to support non-terrestrial networks*, 3GPP Std. TR 38.811, Rev. v15.4.0, Sept. 2020, [Online] Available: https://www.3gpp.org/DynaReport/38811.htm.

[8] A. Alkhateeb, S. Jiang, and G. Charan, "Real-time digital twins: Vision and research directions for 6g and beyond," *IEEE Communications Magazine*, vol. 61, no. 11, pp. 128–134, 2023.

[9] A. Weiss, "Direct position determination of narrowband radio frequency transmitters," *IEEE Signal Processing Letters*, vol. 11, no. 5, pp. 513–516, 2004.

[10] P. Closas and A. Gusi-Amigo, "Direct position estimation of gnss receivers: Analyzing main results, architectures, enhancements, and challenges," *IEEE Signal Processing Magazine*, vol. 34, no. 5, pp. 72–84, 2017.