Multipartite Nonlocality of GHZ States Under Independent Noise Channels

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***Abstract*—The study of multipartite nonlocality provides key insights into the fundamental limits of quantum correlations and their robustness under realistic noise. In this work, we investigate the noisy violation of the Svetlichny inequality for multipartite Greenberger–Horne–Zeilinger (GHZ) states subjected to independent and identically distributed (i.i.d.) dephasing and depolarizing channels. We derive the analytical form of the noisy GHZ state under these channels, and compute the expectation value of the Svetlichny operator for arbitrary number of parties. Specializing to the case of three qubits, we present numerical results showing how noise degrades the violation, and determine the critical noise thresholds beyond which genuine multipartite nonlocality is lost. Our results provide a clear characterization of the relationship between noise models and multipartite quantum correlations.**

***Index Terms*—genuine multipartite entanglement, nonlocality, noisy quantum channel, quantum network, Bell inequality.**

I. Introduction

Multipartite entanglement and nonlocality are central resources for quantum information processing, playing a crucial role in multipartite quantum cryptography [1], especially towards the development of device-independent schemes [2]. Among various tools to characterize genuine multipartite entanglement, the Svetlichny inequality is the most suitable genuine multipartite entanglement witness, capable of distinguishing between partial entanglement and even general form of partial correlation [3]. The Greenberger–Horne–Zeilinger (GHZ)-type states maximally violates this inequality, making it suitable for multiparty cryptogaphy protocols such as quantum conference-key agreement [4].

In realistic scenarios, multipartite GHZ states are distributed through noisy quantum channels [5]. A widely studied model is the independent and identically distributed independent and identically distributed (i.i.d.) noise acting on each subsystem, with depolarizing or dephasing channels being the most representative [6]. Understanding the robustness of Svetlichny inequality violations under such noise models is essential for assessing the feasibility of practical applications of the GHZ states.

In this work, we derive analytical expressions for the expectation value of the Svetlichny operator when GHZ states are distributed through i.i.d. depolarizing and dephasing channels. Our derivation applies to the general M-partite case, allowing us to determine the scaling behavior of nonlocal correlations with system size and noise strength. The derivations results provide an insight on how the noise affect the nonlocality violations as we move from genuine multipartite nonlocality to local separability.

For concreteness, we present explicit numerical results for the case of $M = 3$, which serves as the simplest non-trivial setting. We compute the expectation value of the Svetlichny operator as it decays with increasing noise probability. The corresponding critical noise levels are identified, marking the transition between genuine quantum entanglement and correlations explainable by partially local models. These results highlight the trade-off between multipartite system size, noise resilience, and the possibility of observing violations of the Svetlichny inequality in realistic settings.

II. Preliminaries

A. Quantum states

We now introduce quantum states and their corresponding basis representations. A d-dimensional quantum state in the computational basis is written as a linear combination of its elementary basis $|i\rangle \in \mathcal{H}^d$:

$$|\psi\rangle = \sum_i \lambda_i |i\rangle , \tag{1}$$

with λ_i as complex coefficients and its sum $\sum_i |\lambda_i|^2 = 1$. The statistical description is denoted using the density operator $\mathbf{\Sigma} \in \mathcal{D}(\mathcal{H})$, which is a Hermitian operator with unit trace. A pure state $|\psi\rangle$ is then described as $|\psi\rangle\langle\psi|$, while the general mixed state is given as

$$\mathbf{\Sigma} = \sum_j p_j |\psi_j\rangle\langle\psi_j| , \tag{2}$$

with $\sum_j p_j = 1$ represent the probability of each state $|\psi_j\rangle$. The descriptions of two or more quantum systems is referred to as the composite system. A composite system can be written in the ket form as

$$|\psi\rangle \otimes |\phi\rangle , \tag{3}$$

where both of this state are separable. However, an important composite state that is of our interest is that of entanglement class, in particular the maximally entangled states of M-partite GHZ states defined as:

$$|\mathsf{ghz}-M\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes M} + |1\rangle^{\otimes M}\right), \tag{4}$$

where its density operator form $|\mathsf{ghz}-M\rangle\langle\mathsf{ghz}-M|$ can be written as follows:

$$\mathbf{\Theta}_M = \frac{1}{2}\left(\sum_{i\in\{0,1\}} |i\rangle\langle i|^{\otimes M} + \sum_{i,j\in\{0,1\},i\neq j} |i\rangle\langle j|^{\otimes M}\right). \tag{5}$$

This particular state is called entangled as they cannot be written in the form of tensor product of their individual systems. They exhibit a form of correlation that is non-local, as proven by their violation of the Bell inequality [7].

B. Quantum Channel

In addition, we consider a quantum channel as a linear completely positive trace-preserving (CPTP) map which takes an input density operator $\mathbf{\Theta} \in \mathcal{D}(\mathcal{H}^d)$ to another density operator $\mathcal{N}(\mathbf{\Theta}) \in \mathcal{D}(\mathcal{H}^{d'})$. The depolarizing and dephasing channels are of particular interest, as they represent realistic noisy processes encountered in actual quantum systems [8], [9]. These two noise models are specific instances of the more general Pauli channel. Before introducing the channel, we begin by introducing important unitary operators, namely the pauli matrices as:

$$\begin{aligned}\boldsymbol{\sigma}_\mathrm{x} &= |0\rangle\langle 1| + |1\rangle\langle 0| \\ \boldsymbol{\sigma}_\mathrm{z} &= |0\rangle\langle 0| - |1\rangle\langle 1| \\ \boldsymbol{\sigma}_\mathrm{y} &= \iota\left(|1\rangle|0\rangle - |0\rangle\langle 1|\right),\end{aligned} \tag{6}$$

where $\boldsymbol{\sigma}_\mathrm{y}$ can also be expressed as $\iota\boldsymbol{\sigma}_\mathrm{x}\boldsymbol{\sigma}_\mathrm{z}$. Then the dephasing noise is described by the following map:

$$\mathcal{N}_Z(\mathbf{\Theta}) = (1-q)\mathbf{\Theta} + q\left(\boldsymbol{\sigma}_\mathrm{z}\mathbf{\Theta}\boldsymbol{\sigma}_\mathrm{z}\right), \tag{7}$$

where $q \in [0,1]$. The dephasing noise randomly applies the pauli-z operation with probability q into the input state. The depolarizing noise is described by the following map:

$$\begin{aligned}\mathcal{N}_D(\mathbf{\Theta}) &= (1-q)\mathbf{\Theta} + \frac{q}{2}\boldsymbol{I}_2 \\ &= (1-q)\mathbf{\Theta} + \frac{q}{4}\sum_{k,l\in\{0,1\}} \boldsymbol{\sigma}_\mathrm{x}^k\boldsymbol{\sigma}_\mathrm{z}^l\mathbf{\Theta}\boldsymbol{\sigma}_\mathrm{z}^l\boldsymbol{\sigma}_\mathrm{x}^k,\end{aligned} \tag{8}$$

where $\boldsymbol{I}_2/2 \in \mathcal{D}(\mathcal{H}^2)$ is the maximally mixed state, representing a complete loss of information. So the depolarizing noise cause the input state to completely loss its information with probability q.

C. Svetlichny Inequality

The Svetlichny inequality is a family of multipartite Bell inequality for a dichotomous observables and dimension $d = 2$. For M-partite system, where each particle $i \in \mathbb{Z}_M^+ = \{1, 2, \cdots, M\}$, has two dichotomous observables choice of $\mathcal{A}_i^{(k)}$, $k \in \{0,1\}$, the Svetlichny polynomial can be defined in the recursive form of [3]:

$$\mathcal{S}_M^\pm = \mathcal{S}_{M-1}^\pm\mathcal{A}_M^{(0)} \mp \mathcal{S}_{M-1}^\mp\mathcal{A}_M^{(1)}, \tag{9}$$

where $\mathcal{S}_0^\pm = 1$. Each observables $\mathcal{A}_i^{(k)}$ is defined using its vector form as $\mathcal{A}_i^{(k)} = (\vec{a}_{i,k}\cdot\vec{\sigma})$, with $|\vec{a}_{i,k}|^2 = 1$, and $\vec{\sigma} = \boldsymbol{\sigma}_\mathrm{x}\hat{x} + \boldsymbol{\sigma}_\mathrm{y}\hat{y} + \boldsymbol{\sigma}_\mathrm{z}\hat{z}$ is the pauli vector. The optimal measurement angles for the GHZ state is defined in the x-y plane and hence we can parameterized each observable as $\mathcal{A}_i^{(k)} = \cos(\phi_{i,k})\boldsymbol{\sigma}_\mathrm{x} + \sin(\phi_{i,k})\boldsymbol{\sigma}_\mathrm{y}$. For $M = 2$, the Svetlichny inequality gives the original CHSH inequality and its variant:

$$\begin{aligned}\mathcal{S}_2^+ &= A_1^{(0)}A_2^{(0)} - A_1^{(1)}A_2^{(0)} - A_1^{(0)}A_2^{(1)} - A_1^{(1)}A_2^{(1)}, \\ \mathcal{S}_2^- &= A_1^{(0)}A_2^{(0)} + A_1^{(1)}A_2^{(0)} + A_1^{(0)}A_2^{(1)} - A_1^{(1)}A_2^{(1)}.\end{aligned} \tag{10}$$

The Svetlichny inequality has a property to differentiate between genuine multipartite and partial nonlocality, both for quantum or general nonlocality. For example, a tree-partite quantum state can be a mixture of entangled states described by

$$\mathbf{\Sigma} = \sum_{i,j} p_{i,j}\mathbf{\Sigma}_i^{(1,2)} \otimes \mathbf{\Sigma}_j^{(3)} \tag{11}$$

which consist of bipartite entanglement between the first two party but act locally with the third party. In more general settings, this bipartite entanglement is a specific case of a general correlation, i.e., suppose first and second party are allowed to communicate their measurement results and their observable choice, but act locally with the third party.

Based on the maximum violation of the inequality, the genuine M-partite entanglement has the bound of $|\langle\mathcal{S}_M^\pm\rangle| \leq 2^{M-1}\sqrt{2}$, which is saturated by the GHZ-type states and for other partial nonlocality, it can only reach $|\langle\mathcal{S}_M^\pm\rangle| \leq 2^{M-1}$. For example, in tripartite settings, a mixture of bipartite entanglement cannot achieve expectation value of $4\sqrt{2}$. Even when we allow more general correlation between two parties, it still cannot achieve the maximal bound. Hence maximum violation indicates the existence of a genuine multipartite entanglement.

III. Noisy GHZ under i.i.d. Channel

The parallel i.i.d. channels acting on the GHZ state is described using the notation $\mathcal{N}_\mathcal{X}^{\otimes M}(\mathbf{\Theta}_M)$ as:

$$\frac{1}{2}\left(\sum_{i\in\{0,1\}} \mathcal{N}_\mathcal{X}(|i\rangle\langle i|)^{\otimes M} + \sum_{i,j\in\{0,1\},i\neq j} \mathcal{N}_\mathcal{X}(|i\rangle\langle j|)^{\otimes M}\right). \tag{12}$$

In this noise setting, each subsystem undergoes the same noise independently, which captures the behavior of multipartite entanglement distribution in practical quantum network. Then for each quantum channel, we have the following two propositions as follows.

Proposition 1: For M-partite GHZ state distributed under the i.i.d. dephasing channels with parameter q, we have the noisy state as:

$$\mathcal{N}_Z^{\otimes M}(\mathbf{\Theta}_M) = \frac{1}{2}\left(\sum_{i\in\{0,1\}} |i\rangle\langle i|^{\otimes M} + \eta^M \sum_{i,j\in\{0,1\},\imath\neq j} |i\rangle\langle j|^{\otimes M} \right), \quad (13)$$

with $\eta = (1-2q)$.

Proof: For diagonal case ($i = j$), the dephasing noise leaves the population intact and hence it is simply an identity map. For off-diagonal case ($i \neq j$), we have the single partite map as:

$$\mathcal{N}_Z(|i\rangle\langle j|) = (1-q)|i\rangle\langle j| - q|i\rangle\langle j| = (1-2q)|i\rangle\langle j|, \quad (14)$$

and the M-partite case is simply the tensor products for M-times. □

Proposition 2: For an M-partite GHZ state distributed under the i.i.d. depolarizing channels with parameter q, we have the noisy state as:

$$\mathcal{N}_D^{\otimes M}(\mathbf{\Theta}_M) = \frac{1}{2}\left(\sum_{i\in\{0,1\}} \left(\gamma|i\rangle\langle i| + \frac{q}{2}\boldsymbol{I}_2\right)^{\otimes M} + \gamma^M \sum_{i,j\in\{0,1\},i\neq j} |i\rangle\langle j|^{\otimes M} \right), \quad (15)$$

where $\gamma = (1-q)$.

Proof: For single-partite case, we have the map as:

$$\gamma|i\rangle\langle j| + \frac{q}{4}\sum_{k\in\{0,1\}} \boldsymbol{\sigma}_{\mathrm{x}}^k \left(\sum_{l\in\{0,1\}} \boldsymbol{\sigma}_{\mathrm{z}}^l |i\rangle\langle j| \boldsymbol{\sigma}_{\mathrm{z}}^l \right) \boldsymbol{\sigma}_{\mathrm{x}}^k \quad (16)$$

where for off-diagonal case ($i \neq j$), the left sum become zero and we have

$$\mathcal{N}_D(|i\rangle\langle j|) = \gamma|i\rangle\langle j|, \quad (17)$$

and the M-partite case is simply the tensor product for M-times. For diagonal case, $\boldsymbol{\sigma}_z^l|i\rangle\langle i|\boldsymbol{\sigma}_z^l = |i\rangle\langle i|$, and since the sum is over whole basis, we have:

$$\begin{aligned}\mathcal{N}_D(|i\rangle\langle i|) &= \gamma|i\rangle\langle i| + \frac{q}{2}\sum_{k\in\{0,1\}} \boldsymbol{\sigma}_{\mathrm{x}}^k|i\rangle\langle i|\boldsymbol{\sigma}_{\mathrm{x}}^k \\ &= \gamma|i\rangle\langle i| + \frac{q}{2}\boldsymbol{I}_2, \end{aligned} \quad (18)$$

and hence the M-partite case is also the tensor product for M-times. □

IV. Svetlichny Operator Expectation Value

After deriving the noisy GHZ state, we derive the expectation value of the Svetlichny operator as follows. Since the all observables $\mathcal{A}_i^{(k)}$ is in the x-y plane, for a single system, we have the following relations:

$$\begin{aligned}\langle 0|\mathcal{A}_i^{(k)}|0\rangle &= 0\\ \langle 0|\mathcal{A}_i^{(k)}|1\rangle &= e^{-\iota\phi_{i,k}}\\ \langle 1|\mathcal{A}_i^{(k)}|0\rangle &= e^{\iota\phi_{i,k}}\\ \langle 1|\mathcal{A}_i^{(k)}|1\rangle &= 0, \end{aligned} \quad (19)$$

which indicates that the expectation value only depends on the off-diagonal components. To analyze the local measurement by local observable on each subsystem, we consider the following combined observable

$$\boldsymbol{\mathcal{A}}_M = \mathcal{A}_1 \otimes \mathcal{A}_2 \cdots \otimes \mathcal{A}_M, \quad (20)$$

as the M-partite observables, each with their own parameter ϕ_i, for $i \in \mathbb{Z}_M^+$. The expectation values of this observable using the GHZ states is

$$\begin{aligned}\langle \boldsymbol{\mathcal{A}}_M\rangle &= \frac{1}{2}\left(\sum_{i,j\in\{0,1\}} \langle i|^{\otimes M}\boldsymbol{\mathcal{A}}_M|j\rangle^{\otimes M}\right)\\ &= \frac{1}{2}\left(e^{\iota\boldsymbol{\phi}} + e^{-\iota\boldsymbol{\phi}}\right) = \cos(\boldsymbol{\phi}), \end{aligned} \quad (21)$$

where $\boldsymbol{\phi} = \sum_{i\in\mathbb{Z}_M^+}\phi_i$. Since the elementary maps of the expectation value depends only on the off-diagonal components, then the expectation value given pure GHZ also depends only on the off-diagonal components. For noisy GHZ state, we have the expectation value of the Svetlichny operator $\langle\mathcal{S}_M^{\pm}\rangle$ as:

$$\begin{aligned}\langle\mathcal{S}_M^{\pm}\rangle &= \mathrm{tr}\left(\mathcal{S}_M^{\pm}\mathbf{\Theta}_M(\mathcal{N}_{\mathcal{X}})\right)\\ &= \mathrm{tr}\left(\mathcal{S}_M^{\pm}\mathbf{\Theta}_M^{\mathrm{diag}}(\mathcal{N}_{\mathcal{X}})\right) + \mathrm{tr}\left(\mathcal{S}_M^{\pm}\mathbf{\Theta}_M^{\mathrm{off}}(\mathcal{N}_{\mathcal{X}})\right), \end{aligned} \quad (22)$$

since the diagonal expectation value is zero for the elementary mapping, then its expectation values is also zero. Then the noisy off-diagonal component is simply multiplied by constant factor depending on the noise. The expectation value of the for both noise model can be expressed as:

$$\begin{aligned}\langle\mathcal{S}_M^{\pm}\rangle(\mathcal{N}_Z) &= \eta^M\langle\mathcal{S}_M^{\pm}\rangle_{\mathrm{ghz}},\\ \langle\mathcal{S}_M^{\pm}\rangle(\mathcal{N}_D) &= \gamma^M\langle\mathcal{S}_M^{\pm}\rangle_{\mathrm{ghz}}, \end{aligned} \quad (23)$$

where $\langle\mathcal{S}_M^{\pm}\rangle_{\mathrm{ghz}}$ is the expectation value given pure GHZ. By choosing the parameters as [3]:

$$\begin{aligned}(\phi_{1,0},\phi_{2,0},\cdots\phi_{M,0}) &= (\pm\pi/4, 0, \cdots 0)\\ (\phi_{1,1},\phi_{2,1},\cdots\phi_{M,1}) &= (\pm\pi/4+\pi/2, \pi/2, \cdots\pi/2), \end{aligned} \quad (24)$$

we have that $\langle\mathcal{S}_M^{\pm}\rangle_{\mathrm{ghz}} = 2^{M-1}\sqrt{2}$, and hence we have

$$\begin{aligned}\langle\mathcal{S}_M^{\pm}\rangle(\mathcal{N}_Z) &= \eta^M 2^{M-1}\sqrt{2},\\ \langle\mathcal{S}_M^{\pm}\rangle(\mathcal{N}_D) &= \gamma^M 2^{M-1}\sqrt{2}. \end{aligned} \quad (25)$$

The expectation values for dephasing noise is reduced as a factor of $(1-2q)$ as compared with depolarizing noise with

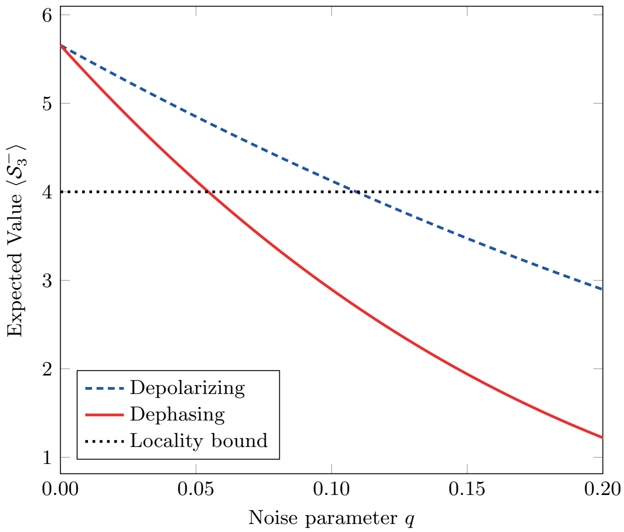

Fig. 1. Svetlichny operator $\mathcal{S}_3^-$ expectation value given noisy GHZ state under dephasing noise $\mathcal{N}_Z^{\otimes M}(\mathbf{\Theta}_M)$ and under depolarizing noise $\mathcal{N}_D^{\otimes M}(\mathbf{\Theta}_M)$. The nonlocality bound is the maximum value of the expectation value that can be achieved without multipartite entanglement.

$(1-q)$ factor. This can be explained from the derivations, that is, the main contributions to the inequality violation is due to the off-diagonal component. When the inequality approach 2^{M-1}, we can no longer distinguish whether the state is a genuine entanglement or only partially entangled. Thus the critical noise threshold is defined as the noise parameter q_c where the expectation value is equal to $2^M - 1$. For dephasing noise, it is

$$q_c(\mathcal{N}_Z) = \frac{1 - 2^{-1/(2M)}}{2} \tag{26}$$

and for depolarizing noise, we have

$$q_c(\mathcal{N}_D) = 1 - 2^{-1/(2M)} \tag{27}$$

V. Numerical Result

We perform numerical calculation of the expectation value of the Svetlichny operator for the case of $M = 3$. The GHZ undergoes the i.i.d. noise for dephasing and depolarizing channel each. For the operator, we choose S_3^- and denote the expectation value of observable $\mathcal{A}_1^{(i)} \otimes \mathcal{A}_2^{(j)} \otimes \mathcal{A}_3^{(k)}$ as $E(i,j,k)$. The explicit form of the operator can be described as:

$$\begin{aligned} &E(0,0,0) + E(1,0,0) + E(0,1,0) + E(0,0,1) \\ &\quad - E(1,1,0) - E(1,0,1) - E(0,1,1) - E(1,1,1), \end{aligned} \tag{28}$$

and the observable for the first parameter is:

$$\mathcal{A}_{1,0} = \frac{\boldsymbol{\sigma}_\mathrm{x} - \boldsymbol{\sigma}_\mathrm{y}}{\sqrt{2}}, \qquad \mathcal{A}_{1,1} = \frac{\boldsymbol{\sigma}_\mathrm{x} + \boldsymbol{\sigma}_\mathrm{y}}{\sqrt{2}}, \tag{29}$$

and for the remaining parameter, they are:

$$\begin{aligned} \mathcal{A}_{2,0} &= \mathcal{A}_{3,0} = \boldsymbol{\sigma}_\mathrm{x}, \\ \mathcal{A}_{2,1} &= \mathcal{A}_{3,1} = \boldsymbol{\sigma}_\mathrm{y}. \end{aligned} \tag{30}$$

Then given the noisy GHZ as described in previous sections, we have the plot of its expected values as a function of noise parameter q in figure 1. Both of the expected value goes down as q increases, suggesting that both noise destroy the nonlocality. The critical noise threshold is obtained numerically as $q_c(\mathcal{N}_Z) = 0.055$ and $q_c(\mathcal{N}_D) = 0.109$.

VI. Conclusions

We have analyzed the robustness of genuine multipartite nonlocality in GHZ states under i.i.d. depolarizing and dephasing noise channels. Starting from the noisy density operator, we derived analytical expressions for the expectation value of the Svetlichny operator for an arbitrary number of parties. This general derivation provides a framework to study the decay of nonlocal correlations under noise.

Focusing on the tripartite case, we presented explicit numerical results illustrating how both depolarizing and dephasing channels reduce the maximal violation of the Svetlichny inequality. The critical noise thresholds, above which genuine multipartite nonlocality is lost, were identified. These results allow a clear distinction between full multipartite nonlocality and partial correlations that can be explained by hybrid local models.

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