Intelligent Reflecting Array Phase Mapping Method with One-Dimensional Coverage Partitioning

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***Abstract*—This study proposes the intelligent reflecting array (IRA) phase mapping method that can directly map the user equipment (UE) position to the IRA phase shift (PS) vector. To construct the mapping between the UE position and the IRA PS vector, we partition the coverage into multiple regions, each with an anchor. The IRA PS vector corresponding to each anchor position is stored offline. The designed mapping is used online to choose the IRA PS vector from the UE's position feedback. Numerical results verified that the IRA phase mapping method with the generalized Lloyd algorithm (GLA)-based partitioning converges to the performance upper bound as the number of regions increases. Also, uniform angle-based partitioning can achieve comparable performance to GLA-based partitioning.**

***Index Terms*—Intelligent reflecting array (IRA), phase mapping method, generalized Lloyd algorithm (GLA).**

I. Introduction

The intelligent reflecting array (IRA) is a one-dimensional (1D) uniform linear array structure of reflecting elements that can flexibly adjust the beam-reflecting direction by controlling each element's phase shift (PS) value. The optimal IRA PS vector can be designed from existing methods utilizing the instantaneous channel state information (CSI) knowledge [1]. However, the instantaneous CSI-based IRA PS vector design introduces significant signaling overheads arising from the complex channel acquisition and IRA PS control signaling. We propose the IRA phase mapping method that can directly map the user equipment (UE) position information feedback to the PS vector, which can reduce the signaling overheads from IRA PS operation. To capture the whole possible analog UE position in the coverage, we define K discrete, non-overlapping regions, each with a representative node called an anchor. By doing this, a finite mapping is possible between the analog UE position represented by an anchor and the PS vector. The coverage partitioning generates K regions and K anchors offline. Then, K IRA PS vectors for each partitioning index are computed and stored in the IRA's memory. During the data transmission (i.e., online phase), the IRA PS vector corresponding to the partitioning index is chosen among the designed IRA PS vectors by referring to the UE's position information feedback. The proposed IRA phase mapping method maximizes the average received signal-to-noise ratio (SNR) by designing the optimal coverage partitioning, which can be interpreted as a K-level quantizer design problem. This study proposes three partitioning strategies: the uniform distance-, uniform angle-, and generalized Lloyd algorithm (GLA)-based partitionings. Simulation results verified that the GLA-based partitioning outperforms others, especially when K is low. As K increases, the IRA phase mapping methods with the GLA- and uniform angle-based partitionings converge to the performance upper bound.

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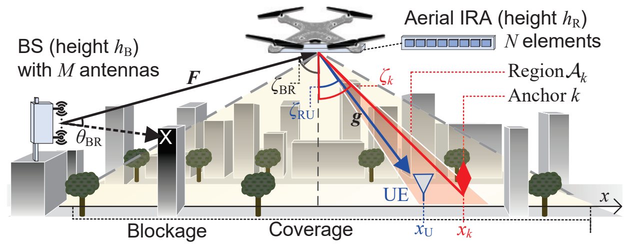

Fig. 1. Aerial IRA-aided downlink communication system models with an M-antenna BS and a single antenna UE in the 1D coverage $\mathcal{A}$. The UE position ζ_{RU} is in the kth region $\mathcal{A}_k$ whose anchor position is given by ζ_k.

II. Aerial IRA-Aided System and Channel Models

This study considers an IRA-aided downlink communication where an M-antenna-equipped base station (BS) supports a single antenna user equipment (UE), as depicted in Fig. 1. Due to the blockages in the urban environments, the direct downlink channel between the BS and UE is assumed to be lost. The N-element IRA is employed using an unmanned aerial vehicle hovering at the height h_R to establish the downlink between the BS and the UE. Here, the IRA is always deployed higher than the height of the BS, i.e., $h_R > h_B$. Considering the maximum IRA beam tilting angle $\zeta_{\max}$, the coverage supported by the aerial IRA is described as $\mathcal{D} = \{x : |x| \leq h_R \tan \zeta_{\max}\}$, or equivalently can be expressed in the angular domain as $\mathcal{A} = \{\zeta : |\zeta| \leq \zeta_{\max}\}$[1]. The position of UE is given by $x_U = h_R \tan \zeta_{RU}$ where $\zeta_{RU} \in \mathcal{A}$ is the angle of departure (AoD) from IRA to UE. Similarly, the position of

[1]Here, the communication coverage is regarded as a 1D street in the urban canyon, where the width is negligible compared to the length of the coverage. The system model can directly be extended to the two-dimensional coverage.

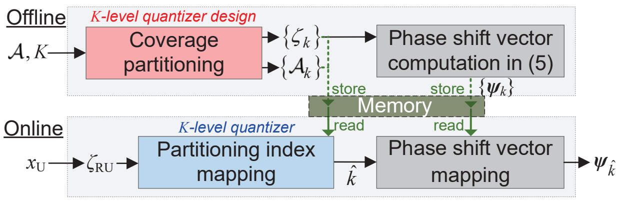

Fig. 2. Proposed IRA phase mapping method with offline and online phases. In the offline phase, the coverage regions $\{\mathcal{A}_k\}$ and the corresponding anchor positions $\{\zeta_k\}$ are designed in coverage partitioning. Then, the PS vectors $\{\boldsymbol{\psi}_k\}$ are computed and stored in the IRA's memory. In the online phase, the UE position ζ_{RU} is mapped into the partitioning index $\hat{k}$, and the corresponding PS vector is chosen from the stored $\{\boldsymbol{\psi}_k\}$.

BS is given by $x_{\mathrm{B}} = (h_{\mathrm{R}} - h_{\mathrm{B}}) \tan \zeta_{\mathrm{BR}}$ where ζ_{BR} denotes the angle of arrival from BS to IRA. Also, θ_{BR} denotes the AoD from BS to IRA. This study assumes that the BS and IRA are located in fixed positions, and the UE position, described in angular domain ζ_{RU}, is distributed uniformly in $\mathcal{A}$.

Since the line-of-sight (LoS) path is dominant in the air-to-ground channels, the channel between the BS and aerial IRA $\boldsymbol{F} \in \mathbb{C}^{N \times M}$, and the channel between the aerial IRA and the UE $\boldsymbol{g} \in \mathbb{C}^{N \times 1}$ can be modeled, respectively, as follows [2]:

$$\boldsymbol{F} = \sqrt{\eta(d_{\mathrm{BR}})} e^{-j2\pi d_{\mathrm{BR}}/\lambda} \boldsymbol{a}_{\mathrm{R}}(\zeta_{\mathrm{BR}}) \boldsymbol{a}_{\mathrm{B}}^{H}(\theta_{\mathrm{BR}}), \tag{1a}$$
$$\boldsymbol{g} = \sqrt{\eta(d_{\mathrm{RU}})} e^{-j2\pi d_{\mathrm{RU}}/\lambda} \boldsymbol{a}_{\mathrm{R}}(\zeta_{\mathrm{RU}}), \tag{1b}$$

where $\eta(d)$ is the distance-dependent path loss model; $d_{\mathrm{BR}} = \frac{h_{\mathrm{R}} - h_{\mathrm{B}}}{\cos \zeta_{\mathrm{BR}}}$ is the distance between the BS and IRA; and $d_{\mathrm{RU}} = \frac{h_{\mathrm{R}}}{\cos \zeta_{\mathrm{RU}}}$ is the distance between the IRA and UE. Also, $\boldsymbol{a}_{\mathrm{B}}(\theta) \in \mathbb{C}^{M \times 1}$ is the uniform linear array steering vector of the BS, whose mth element is given by $e^{-\frac{j2\pi\Delta}{\lambda}(m-1)\sin\theta}$ where λ and Δ are the signal wavelength and the antenna (or IRA element) spacing, respectively. Similarly, the IRA steering vector is denoted by $\boldsymbol{a}_{\mathrm{R}}(\zeta) \in \mathbb{C}^{N \times 1}$ whose nth element is given by $e^{\frac{j2\pi\Delta}{\lambda}(n-1)\sin\zeta}$ [2].

The received symbol $r \in \mathbb{C}$ at the UE is written as follows:

$$r = \boldsymbol{g}^{H} \operatorname{diag}(\boldsymbol{\psi}) \boldsymbol{F} \boldsymbol{w} x + z, \tag{2}$$

where $\boldsymbol{w} \in \mathbb{C}^{M \times 1}$ is the transmit precoder satisfying $\|\boldsymbol{w}\| = 1$; x is the information symbol satisfying $\mathrm{E}\left[|x|^2\right] = P$ where P is the transmit power of BS; z is the additive white Gaussian noise with zero mean and σ^2 variance; and $\boldsymbol{\psi} = \left[\psi_1 \cdots \psi_n \cdots \psi_N\right]^T \in \mathbb{C}^{N \times 1}$ satisfying $|\psi_n| = 1, \forall n$.

Using the maximum ratio transmission at the BS, i.e., $\boldsymbol{w} = \frac{\boldsymbol{F}^H \operatorname{diag}(\boldsymbol{\psi})^H \boldsymbol{g}}{\|\boldsymbol{F}^H \operatorname{diag}(\boldsymbol{\psi})^H \boldsymbol{g}\|}$, the received SNR at the UE position ζ_{RU} is written as $\mathrm{SNR}(\boldsymbol{\psi}, \zeta_{\mathrm{RU}}) = P\sigma^{-2}\gamma(\boldsymbol{\psi}, \zeta_{\mathrm{RU}})$. Here, $\gamma(\boldsymbol{\psi}, \zeta)$ is an effective channel gain obtained by using PS vector $\boldsymbol{\psi}$ at UE position ζ, which can be derived from (1) and (2) as follows:

$$\gamma(\boldsymbol{\psi}, \zeta) = \eta(d_{\mathrm{BR}}) \eta\left(\tfrac{h_{\mathrm{R}}}{\cos\zeta}\right) M \left|\boldsymbol{\psi}^T \boldsymbol{a}_{\mathrm{R}}^{*}(\zeta) \odot \boldsymbol{a}_{\mathrm{R}}(\zeta_{\mathrm{BR}})\right|^2, \tag{3}$$

where $\odot$ is the element-wise product operator.

III. Proposed IRA Phase Mapping Method

As shown in Fig. 2, the proposed IRA phase mapping method consists of offline and online phases. In the offline phases, the coverage $\mathcal{A}$ is partitioned into K regions, and each region has a representative node called an anchor. Specifically, the kth region is denoted by $\mathcal{A}_k$ which satisfies $\cup_{k=1}^{K} \mathcal{A}_k = \mathcal{A}$, $\cap_{k=1}^{K} \mathcal{A}_k = \emptyset$; and corresponding kth anchor position in angular domain is denoted by ζ_k (see Fig. 1). Then, the IRA controller stores the set $\{\boldsymbol{\psi}_k\}$ where $\boldsymbol{\psi}_k$ denotes the IRA PS vector that maximizes the received SNR when UE is located at ζ_k. During the online phases, the IRA receives feedback signals containing the UE position ζ_{RU}. Next, the partitioning index $\hat{k}$ is identified using the mapping between ζ_{RU} and $\zeta_{\hat{k}}$, designed in the offline phases. Finally, referring to the partitioning index $\hat{k}$, the IRA selects $\boldsymbol{\psi}_{\hat{k}}$ from memory.

A. *Offline Phase: Coverage Partitioning and PS Vector Design*

For coverage partitioning in the offline phases, two simple approaches, namely uniform distance and uniform angle-based partitionings, can be considered. The kth anchor positions following the uniform distance and uniform angle-based partitioning are written, respectively, as follows:

$$\zeta_{\mathrm{a},k} = -\zeta_{\max} + \tfrac{(2k-1)\zeta_{\max}}{K}, \tag{4a}$$
$$\zeta_{\mathrm{d},k} = \tan^{-1}\left(\left(\tfrac{2k-1}{K} - 1\right)\tan\zeta_{\max}\right), \tag{4b}$$

for $k \in \{1, \ldots, K\}$. The kth region $\mathcal{A}_{\mathrm{a},k}$ and $\mathcal{A}_{\mathrm{d},k}$ are constructed by mapping every $\zeta \in \mathcal{A}$ to their anchor which is the closest in angle and distance, respectively. Given the anchor position ζ_k, the IRA PS vector that maximizes the received SNR can be derived from (3) as follows:

$$\boldsymbol{\psi}(\zeta_k) = \boldsymbol{a}_{\mathrm{R}}(\zeta_k) \odot \boldsymbol{a}_{\mathrm{R}}^{*}(\zeta_{\mathrm{BR}}). \tag{5}$$

The performance of the IRA phase mapping method can be improved by solving the optimal coverage partitioning problem, which is formulated as follows:

$$\underset{\{\zeta_k\},\{\mathcal{A}_k\}}{\arg\max} \sum_{k=1}^{K} p_k \mathrm{E}_{\zeta}\left[\gamma\left(\boldsymbol{\psi}\left(\zeta_k\right), \zeta\right) | \zeta \in \mathcal{A}_k\right] \tag{6a}$$
$$\text{s.t.} \quad \cup_{k=1}^{K} \mathcal{A}_k = \mathcal{A}, \ \cap_{k=1}^{K} \mathcal{A}_k = \emptyset. \tag{6b}$$

Here, $p_k \triangleq \int_{\mathcal{A}_k} f_Z(\zeta) d\zeta$ where $f_Z(\zeta)$ is the probability density function of ζ, and $\gamma(\boldsymbol{\psi}_k, \zeta)$ is the effective channel gain obtained at the position ζ with IRA PS vector $\boldsymbol{\psi}_k$, which can be derived by substituting (5) into (3) as

$$\gamma(\boldsymbol{\psi}(\zeta_k), \zeta) = \eta(d_{\mathrm{BR}}) \eta\left(\tfrac{h_{\mathrm{R}}}{\cos\zeta}\right) M N^2 \left|\mathrm{Diric}_N(\sin\zeta - \sin\zeta_k)\right|^2, \tag{7}$$

where the detailed derivation is omitted. Here, $\mathrm{Diric}_n(x)$ is the normalized Dirichlet sinc function defined as $\mathrm{Diric}_n(x) \triangleq \frac{\sin(nx/2)}{n\sin(x/2)}$ for integer n. The problem (6) can be interpreted as a K-level quantizer design problem that maps the continuous angle set $\mathcal{A}$ into the discrete angle set $\{\zeta_k\}$ to maximize the average received SNR [3]. However, the problem is intractable due to the highly non-convex features of (6a) and (6b).

To efficiently handle (6), this study employs the GLA, a well-known quantizer design strategy, which is widely used for data compression and clustering [4]. The GLA is an iterative algorithm where each iteration consists of two subproblems: i) finding the optimal regions under given anchor positions in

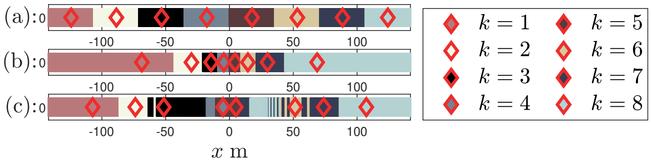

Fig. 3. Examples of coverage partitioning for IRA phase mapping system when $K = 8$, $N = 100$, $M = 4$, and $h_{\mathrm{R}} = 25$ m. Marker '◇' indicates the positions of the anchor. (a) Uniform distance, (b) Uniform angle, (c) GLA.

the previous iteration and ii) optimal anchor position searching with fixed regions obtained from the first subproblem. Specifically, suppose that $\zeta_k^{(i-1)}$ fixed, the set of K regions that achieves the maximum average effective channel gain is obtained in the first subproblem of the ith iteration, which is written as follows:

$$\left\{\mathcal{A}_k^{(i)}\right\} = \underset{\{\mathcal{A}_k\}}{\arg\max} \sum_{k=1}^{K} p_k \mathrm{E}_\zeta\!\left[\gamma\!\left(\boldsymbol{\psi}\left(\zeta_k^{(i-1)}\right), \zeta\right)\middle|\, \zeta \in \mathcal{A}_k\right], \tag{8}$$

subject to (6b). The initial (i=0) anchor positions follow the uniform angle-based partitioning in (4a). Then, by fixing $\mathcal{A}_k^{(i)}$, the kth anchor position at the ith iteration is chosen as follows:

$$\zeta_k^{(i)} = \underset{\zeta_k}{\arg\max}\ \mathrm{E}_\zeta\left[\gamma\left(\boldsymbol{\psi}\left(\zeta_k\right), \zeta\right)\middle|\, \zeta \in \mathcal{A}_k^{(i)}\right]. \tag{9}$$

Here, the conditional expectation operators in (8) and (9) are approximately computed using the numerical integral methods. By repeating two subproblems until convergence, the optimal $\{\zeta_{1,k}\}, \{\mathcal{A}_{1,k}\}$ can be obtained. The Lloyd iteration is said to converge when the difference of (6a) in the current and previous iterations is smaller than the stopping threshold ϵ. Fig. 3 illustrates the examples of the uniform distance, uniform angle, and GLA-based coverage partitionings when $K = 8$.

B. Online Phase: Partitioning Index and PS Vector Mappings

From the UE position information feedback, x_{U}, the IRA calculates the angle domain UE position as $\zeta_{\mathrm{RU}} = \arctan \frac{x_{\mathrm{U}}}{h_{\mathrm{R}}}$. Then, the partitioning index $\hat{k}$ is obtained from the coverage partitioning mapping from ζ_{RU} to $\zeta_{\hat{k}}$ (i.e., a K-level quantization), designed in offline. Finally, the PS vector $\boldsymbol{\psi}_{\hat{k}}$ is directly mapped from the stored set $\{\boldsymbol{\psi}_k\}$ using $\hat{k}$.

IV. Numerical Results

This section evaluates the performance of the IRA phase mapping methods with three coverage partitionings: uniform distance, uniform angle, and GLA-based partitionings. The simulation parameters are listed as follows: the number of BS antennas $M = 4$; carrier frequency $f_c = 2.5$ GHz; $\Delta = \frac{\lambda}{2}$; $\sigma^2 = -104$ dBm; $\zeta_{\max} = 80°$; $\zeta_{\mathrm{BR}} = -84°$; and the height of BS and aerial IRA $h_{\mathrm{B}} = 10$ m and $h_{\mathrm{R}} = 25$ m, respectively. The position of UE is uniform-randomly generated in the coverage $\mathcal{D}$. The path loss model is given as $\eta(d) = -28 - 20\log_{10}(f_c) - 22\log_{10}(d)$ following the 3GPP standard [5]. The stopping threshold of GLA is set to $\epsilon = 10^{-30}$.

In Fig. 4, the received SNR performance of various IRA phase mapping methods is evaluated for the number of regions

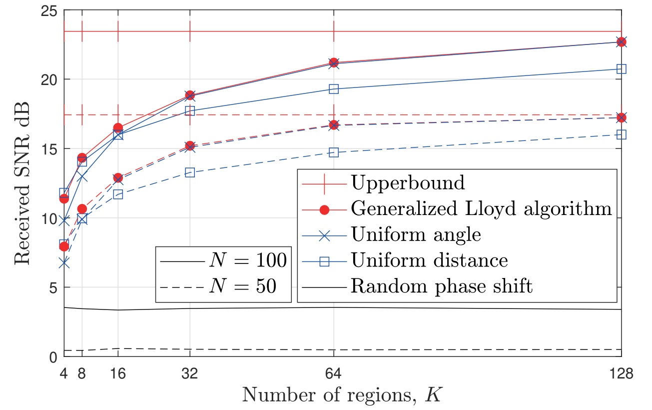

Fig. 4. Received SNR over the number of regions, K, for $N \in \{50, 100\}$ when $P = 30$ dBm.

K, when $N \in \{50, 100\}$ and $P = 30$ dBm. Here, the performance upper bound is when $K \to \infty$, i.e., the position error between the UE and the corresponding anchor is always zero. The numerical results verified that the performance of the IRA phase mapping methods increases as K increases. When $K < 32$, the performance gap between the GLA and the uniform angle-based partitioning exists; on the other hand, the performance of the IRA phase mapping methods using the GLA and uniform angle-based partitioning is almost identical for $K \geq 32$, converging to their upper bound as K increases. On the other hand, the uniform distance-based partitioning exhibits a significant performance gap between the upper bound even when K is high.

V. Conclusion

This study proposed the IRA phase mapping method that directly chooses the IRA PS vector from the UE's position feedback. Uniform distance, uniform angle, and GLA-based coverage partitionings were proposed to map the UE position and the PS vector. The PS vector is obtained by mapping the UE position and the corresponding PS vector. Numerical results confirmed that the GLA- and uniform angle-based partitionings converge to the SNR performance upper bound as the number of regions increases.

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