

Direction of Arrival Estimation Through Joint Phase-Time Arrays

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Abstract—In recent years, private 5G, which allows municipalities and companies to build their own 5G networks in specific areas, such as within their premises, has been attracting attention. Multiple base stations (BSs) are expected to be operated in the vicinity as private 5G spreads more in the future. In such a scenario, it is essential to suppress and avoid mutual interference between neighboring operators. For interference suppression, the accurate estimation of the direction-of-arrival (DoA) of the interfering signal is necessary. This paper proposes a DoA estimation method using the joint phase-time array (JPTA), a type of analog beamforming (ABF). Computer simulation results show that the proposed DoA estimation method can estimate the DoA with the accuracy of 2° at a 90-percentile value.

Index Terms—Local 5G, joint phase-time arrays, direction-of-arrival estimation

I. INTRODUCTION

Private 5G, which allows companies and municipalities to build their own 5G networks in limited areas, is gaining popularity worldwide [1]. In Japan, private 5G is widely known as local 5G. Local 5G allocates up to 300 MHz bandwidth in the Sub-6 band and up to 900 MHz bandwidth in the millimeter wave band for enterprises and municipalities, and can provide high data rate communication service within the area where it is deployed, e.g., inside the deployer's premise [2]. However, the large channel propagation loss at high frequency bands shortens the communication range. Therefore, beamforming (BF), which forms a directional beam towards the desired direction by using a large number of antenna elements, is essential to compensate for channel propagation loss by providing BF gain in a specific direction.

Since local 5G is still not yet fully deployed, each base station (BS) can have a sufficient separation distance from each other to avoid mutual interference. However, multiple local 5G BSs are expected to operate in the neighborhood in the near future. In such a situation, it may become difficult to guarantee sufficient separation between the neighboring BSs. Thus, it is necessary to consider interference suppression and avoidance [2]. Furthermore, since neighboring BS belong to the different local 5G operators, it is not appropriate to have tight cooperation or collaboration between them. Thus, such

interference suppression and avoidance shall be individually performed by each operator.

Direction-of-arrival (DoA) estimation of interfering signals is important for suppressing and removing interference from other systems. There are many sophisticated DoA estimation methods. The beamformer and Capon methods have been proposed as techniques for estimating the DoA of interfering signals [3]. These methods estimate the DoA of an interfering signal by comparing the received power at each angle. However, due to analog beam constraints, these methods require the main lobe to be swept in all directions by switching the beam, which incurs overhead each time the DoA of the interference is estimated and reduces the effective throughput of the system. To direct the gain in multiple directions in a single time slot, digital beamforming, which relies on multiple radio frequency (RF) chains, is beneficial. However, it consumes more power and costs more.

This paper proposes a DoA estimation method that utilizes an analog beamforming (ABF) structure called joint phase-time arrays (JPTA) [4]. The JPTA consists of true time delay (TTD) elements and phase shifters. The TTD element adds a time delay to the signal received at each antenna. This allows beam training to be performed using a single orthogonal frequency division multiplexing (OFDM) symbol. In other words, JPTA can generate a single beam whose directivity varies with the frequency of the signal. Therefore, generating a beam with JPTA over the system bandwidth enables the system to estimate the DoA of an interfering signal without sweeping the beam. The proposed DoA estimation method operates based on the relationship between frequency and gain directivity of the beam behavior created by the JPTA. A summary of our contributions is given below. We derive the solution for DoA estimation using JPTA based on the relationship between the frequencies with high received power and BF weights, and show the algorithm for DoA estimation. From the computer simulation results, we show that the proposed algorithm can estimate the DoA accurately. Furthermore, how the accuracy of DoA estimation differs depending on the number of antennas is evaluated.

The remainder of this paper is organized as follows. Section II briefly explains the system model considered in this paper. Section III explains the core technology, i.e., JPTA, used by the proposed DoA estimation method. Section IV explains the proposed DoA estimation method. The numerical evaluation

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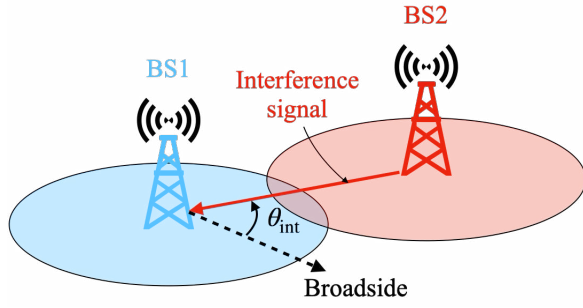


Fig. 1: System environment considered in this paper.

results are provided in Section V. The paper is concluded by Section VI.

II. SYSTEM MODEL

This paper assumes an environment where two different local 5G systems coexist in their vicinity, as shown in Fig. 1. The DoA of interference signals from BS2 at BS1 is assumed to be θ_{int} [rad], where the broadside direction of the antenna at BS1 is defined as $\theta_{\text{int}} = 0$. Both systems use the frequency bandwidth of W [Hz] with center frequency f_0 [Hz]. Since it is essential to use the massive antenna array to enhance the system throughput while combating the high propagation loss due to high-frequency band, each BS is assumed to be equipped with $M \gg 1$ antenna elements. Since both systems share the same frequency bandwidth, there is possible mutual interference at the boundary of each system. Thus, it is necessary to avoid the interference by some means. Since we consider the scenario with two local 5G systems, each belonging to a different operator, it is not appropriate to assume tight cooperation or coordination among the systems. Thus, each system needs to handle the interference from its counterpart independently. For such interference management, it is essential to obtain the DoA.

A. Joint Phase Time Array Architecture

The JPTA architecture is shown in Fig. 2, where BS is equipped with a uniform linear array (ULA) of M (set $\mathcal{M} = \{1, 2, \dots, M\}$) antenna elements equally spaced in a straight line that are connected to one RF chain [4]. The antenna spacing is half the wavelength of the center frequency f_0 [Hz]. There are N TTD elements (set $\mathcal{N} = \{1, 2, \dots, N\}$). Each TTD element is connected to one or more than one antenna element via mapping matrix $\mathbf{P} \in \{0, 1\}^{M \times N}$, as shown in Fig. 2.

For any TTD element $n \in \mathcal{N}$, set $\mathcal{M}_n = \{m \in \mathbb{N} | \frac{(n-1)M}{N} < m \leq \frac{nM}{N}\}$ denotes the antenna elements that are connected to TTD element n . The delay amount, τ [sec], of each TTD element is assumed to be adjustable within the range of $0 \leq \tau \leq M/W$. The weight of the antenna element m has a unit amplitude with the adjustable phase in the range of $-\pi \leq \phi_m \leq \pi$ [rad].

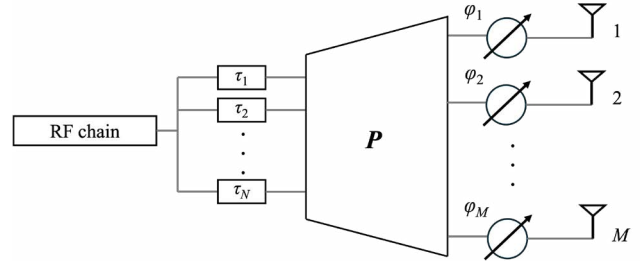


Fig. 2: The JPTA architecture

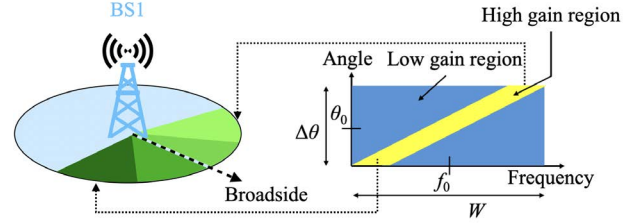


Fig. 3: Beam behavior

This paper adopts OFDM with K subcarriers. The downlink signal transmitted on the subcarrier $k \in \mathcal{K} = \{\lfloor \frac{1-K}{2} \rfloor, \dots, \lfloor \frac{K-1}{2} \rfloor\}$ is expressed as follows.

$$\begin{aligned} \mathbf{x}_k &= \frac{1}{\sqrt{M}} \underbrace{\begin{bmatrix} e^{j\phi_1} & 0 & \dots & 0 \\ 0 & e^{j\phi_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{j\phi_M} \end{bmatrix}}_{\triangleq \mathbf{T}} \mathbf{P} \underbrace{\begin{bmatrix} e^{-j2\pi f_k \tau_1} \\ e^{-j2\pi f_k \tau_2} \\ \vdots \\ e^{-j2\pi f_k \tau_N} \end{bmatrix}}_{\triangleq \mathbf{d}_k} \times \alpha_k s_k \\ &= \mathbf{T} \mathbf{P} \mathbf{d}_k \alpha_k s_k, \end{aligned} \quad (1)$$

where $\alpha_k \in \mathbb{C}$ and $s_k \in \mathbb{C}$ denote the digital BF weights and digitally modulated data symbol of the subcarrier k , f_k [Hz] is the frequency of the subcarrier k , τ_n is the delay of TTD element n , ϕ_m is the phase of the analog weight at the antenna element m . In this JPTA structure, the diagonal matrix \mathbf{T} of $M \times M$ represents the effect of the analog beamformer, and \mathbf{d}_k of $N \times 1$ represents the effect of the phase change introduced by the TTD element. Thus, the unit-norm ABF weights in the downlink signal of the subcarrier k are $\mathbf{T} \mathbf{P} \mathbf{d}_k$. The set of unit-norm ABF weights $\mathbf{c}_k = \mathbf{T} \mathbf{P} \mathbf{d}_k$ obtained by JPTA is denoted as $\mathcal{C} = \{\mathbf{c}_k | k \in \mathcal{K}\}$.

III. BEAMFORMING DESIGN

This paper considers a beam behavior for estimating the DoA of the interfering signal. The set of unit-norm analog BF weights of JPTA, \mathcal{C} , is obtained through the optimization algorithm [4].

A. Beam Behavior and Its Applications

In the considered beam behavior, the BF set \mathcal{C} is designed so that the high gain region varies linearly with signal frequency f over angle θ , as shown in Fig. 3. Thus, the angle $\theta(f)$ at frequency f in the high gain region varies linearly over the bandwidth W . This beam behavior is useful for estimating the DoA of interfering signals from other systems in order to prevent inter-cell interference in the environment considered in this paper. Conventional methods of estimating the DoA of signals, such as the beamformer and Capon methods require large computational complexity and may not be able to obtain the desired information due to the large pathloss [5]. If the BS sweeps the analog beams to estimate the DoA of the interfering signal, it causes overhead due to beam switching. In contrast, in the considered beam behavior, each frequency component has strong antenna gain to specific directions. Therefore, the DoA of the interfering signal is obtained by comparing the received signal power at each frequency. Since the BS sweeps the beam in the frequency domain, it allows a large area to be searched at once, reducing the overhead of beam switching.

The set of BF weight vectors at subcarrier k is denoted as $\mathcal{B} = \{\mathbf{b}_k | k \in \mathcal{K}\}$ and the total power constraint is expressed as $\sum_{k \in \mathcal{K}} \|\mathbf{b}_k\|^2 \leq P_{\text{sum}}$. For realizing the desired beam pattern, we set $\mathcal{B} = \{\mathbf{b}_k | k \in \mathcal{K}\}$ as follows

$$\mathbf{b}_k = \sqrt{\frac{P_{\text{sum}}}{MK}} \mathbf{a}_k \left(\theta_0 + k \frac{\Delta\theta}{K} \right), \quad (2)$$

where $\mathbf{a}_k \in \mathcal{C}^{M \times 1}$ is the array response vector for a given angle $-\pi/2 \leq \theta \leq \pi/2$ at the BS, which is given by

$$\begin{aligned} \mathbf{a}_k(\theta) \\ = \left[1 \quad \exp\left(j \frac{\pi \sin(\theta) f_k}{f_0}\right) \quad \dots \quad \exp\left(j \frac{(M-1) \pi \sin(\theta) f_k}{f_0}\right) \right]^T. \end{aligned} \quad (3)$$

B. Algorithm for Obtaining BF Weights to Realize Desired Beam Behavior [4]

Obtaining the unit-norm ABF weights $\mathbf{c}_k = \mathbf{TPd}_k$ of the JPTA to realize the BF weights \mathcal{B} requires the minimization of the following cost function:

$$\begin{aligned} \tilde{\mathcal{F}}_{\text{obj}}(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\phi}, \mathcal{B}) = \sum_{k \in \mathcal{K}} \frac{1}{K} \left[(\|\mathbf{b}_k\| - |\alpha_k|)^2 \right. \\ \left. + \left\| \frac{\mathbf{b}_k}{\|\mathbf{b}_k\|} - \mathbf{TPd}_k e^{j\angle\alpha_k} \right\|^2 \right]. \end{aligned} \quad (4)$$

Assuming $\|\mathbf{b}_k\| = |\alpha_k|$ and focusing on the second term of Eq. (4), the parameters of JPTA BF weights that minimize $\tilde{\mathcal{F}}_{\text{obj}}$ can be obtained as

$$\{\boldsymbol{\tau}^*, \boldsymbol{\phi}^*, \angle\boldsymbol{\alpha}^*\} = \underset{\boldsymbol{\tau}, \boldsymbol{\phi}, \angle\boldsymbol{\alpha}}{\text{argmax}} \left\{ \sum_{k \in \mathcal{K}} \text{Re} \left[e^{j\angle\alpha_k} \bar{\mathbf{b}}_k^\dagger \mathbf{TPd}_k \right] \right\}, \quad (5)$$

where $\bar{\mathbf{b}}_k \triangleq \mathbf{b}_k / \|\mathbf{b}_k\|$, $\angle\boldsymbol{\alpha} \triangleq \{\angle\alpha_k | k \in \mathcal{K}\}$ and $(\cdot)^\dagger$ denotes the Hermitian transpose operation. In the following, we will optimize $\boldsymbol{\alpha}$, $\boldsymbol{\tau}$, $\boldsymbol{\phi}$ for Eq. (5) in the range of $-\pi \leq \alpha_k \leq \pi$, $0 \leq \tau_n \leq \kappa/W$, $-\pi \leq \phi_m \leq \pi$. However, it is difficult to find

the jointly optimal solution of $\boldsymbol{\alpha}$, $\boldsymbol{\tau}$, and $\boldsymbol{\phi}$ of Eq. (5) at once. Therefore, an alternating optimization approach is adopted to find the conditionally optimal $\angle\boldsymbol{\alpha}$ for a given $\{\boldsymbol{\tau}, \boldsymbol{\phi}\}$ and vice versa [4]. Several iterations of this optimization yield an approximation of the JPTA BF weights to the desired BF weights.

Step 1 With $\alpha_k = 0$, find the optimal solution of the following equation:

$$\tau_n^*(\angle\boldsymbol{\alpha}) = \underset{\tau_n}{\text{argmax}} \left\{ \sum_{m \in \mathcal{M}_n} \left| \sum_{k \in \mathcal{K}} e^{j\angle\alpha_k} [\bar{\mathbf{b}}_k]_m^* e^{-j2\pi f_k \tau_n} \right| \right\}, \quad (6)$$

where $(\cdot)^*$ denotes complex conjugate operation. We find the optimal solution in the range $0 \leq \tau \leq M/W$ in brute-force search with a step size of $\Delta\tau$ [sec].

Step 2 The optimal solution ϕ_m^* for τ_n^* is obtained by

$$\phi_m^*(\angle\boldsymbol{\alpha}) = \angle \left[\sum_{k \in \mathcal{K}} \omega_k e^{j\angle\alpha_k} [\bar{\mathbf{b}}_k]_m^* e^{j2\pi f_k \tau_n^*} \right]. \quad (7)$$

Step 3 The optimal solution α^* for τ_n^* , ϕ_m^* obtained by Eqs. (6) and (8) is obtained as follows

$$\angle\alpha_k^* = \angle \left[\sum_{n=1}^N \sum_{m \in \mathcal{M}_n} [\bar{\mathbf{b}}_k]_m e^{-j\phi_m} e^{j2\pi f_k \tau_n^*} \right]. \quad (8)$$

By iterating these steps for a fixed number of iterations I_{max} , the JPTA approximation to the desired beamformer can be obtained.

IV. PROPOSED METHOD

This paper proposes a method for estimating the DoA of interfering signals using the JPTA beam behavior, given in Section III-A. As described in Section III-B, the optimized JPTA unit-norm ABF weight set \mathcal{C} allows us to generate the BF with a gain for a different direction θ at different frequency. In the case of ideal JPTA BF, given by Eq. (2), there is a following relationship between the subcarrier index k and the corresponding beam direction θ :

$$\theta = \Theta(k) = \theta_0 + k \frac{\Delta\theta}{K}, \quad (9)$$

or

$$k = \Theta^{-1}(\theta) = (\theta - \theta_0) \cdot \frac{K}{\Delta\theta}, \quad (10)$$

where $\Theta(\cdot)$ is a one-to-one projection function. Thus, suppose the signal \mathbf{y}_k is impinging from direction θ_{int} , the power of the received signal \mathbf{y}_k after passing through the JPTA BF weights \mathcal{C} is large at the $k_{\text{int}} (= \Theta^{-1}(\theta_{\text{int}}))$ th subcarrier corresponding to the DoA θ_{int} and smaller for other subcarriers.

Therefore, the received signal power at signal frequency f_k is given by

$$|\mathbf{y}_k^T \cdot \mathbf{TPd}_k|^2 \begin{cases} \neq 0 & \text{for } k = \Theta^{-1}(\theta_{\text{int}}) \\ \approx 0 & \text{otherwise} \end{cases}, \quad (11)$$

where $(\cdot)^T$ denotes the transpose operation.

The above observation means that obtaining index k^* that gives the largest signal power indicates the DoA of the

TABLE I: Parameter definition and default values.

| Parameters | Values |
|-------------------------------------|---------------------|
| Center frequency, f_0 | 4.85 GHz |
| Bandwidth, W | 40 MHz |
| Number of subcarriers, K | 2592 |
| Number of resource blocks, R | 216 |
| Frequency of subcarriers, f_k | $4.85 + 0.1k/K$ GHz |
| Number of RF chain, N_{RF} | 1 |
| Number of antennas, M | 16 or 64 |
| Number of TTDs, N | $N = M$ |

interfering signal by using Eq. (9). In the following, we describe the algorithm for estimating k^* using the array gain for the arriving signal from angle θ with the generated JPTA BF weight \mathbf{TPd}_k at the k th frequency. By setting $k = k^*$ in Eq. (9), the DoA of a signal from another system can be estimated.

Step 1 The resource block (RB) that gives the maximum signal power is obtained. The basic unit of resource allocation in 5G communications is the RB, which consists of 12 OFDM subcarriers. This paper assumes that all 12 subcarriers in the RB are used. Let \mathbf{y}_k be the received signal at the k th subcarrier, the RB with the largest signal power contained in the signals of other systems is estimated by

$$r^* = \underset{r \in \mathcal{R}}{\operatorname{argmax}} \sum_{k \in \mathcal{K}_r} |\mathbf{y}_k^T(\theta_{\text{int}}) \cdot \mathbf{TPd}_k|^2, \quad (12)$$

where $r \in \mathcal{R} = \{1, 2, \dots, R\}$ is the r th RB and \mathcal{K}_r is the set of subcarriers within the r th RB.

Step 2 The center subcarrier of the r^* th RB, k^* , is obtained by

$$k^* = \left\lfloor \frac{1-K}{2} \right\rfloor + 12 \times r^* + 6. \quad (13)$$

Step 3 Based on the relationship between the index of the subcarrier of the arriving signal and the DoA, which is given in Eq. (9), the DoA of the interfering signal θ^* is obtained by

$$\theta^* = \Theta(k^*) = \theta_0 + \frac{k^*}{K} \Delta\theta \quad (14)$$

V. SIMULATION RESULTS

This section provides numerical evaluation results of the proposed DoA estimation method described in Section IV. Simulation parameters are listed in Table I. The center frequency is set to $f_0 = 4.85$ GHz [2] and the bandwidth is $W = 40$ MHz. The number of subcarriers is set to $K = 2592$ and index k is defined as $\lfloor (1-K)/2 \rfloor \leq k \leq \lfloor (K-1)/2 \rfloor$. The number of antenna, M , is set to 16 or 64 and number of TTDs N is set to $N = M$ and the number of RF chains is $N_{\text{RF}} = 1$. The unit-norm analog JPTA BF set \mathcal{C} are obtained as the algorithm described in Section III with $I_{\text{max}} = 10$, which provides efficient convergence property as indicated by [4].

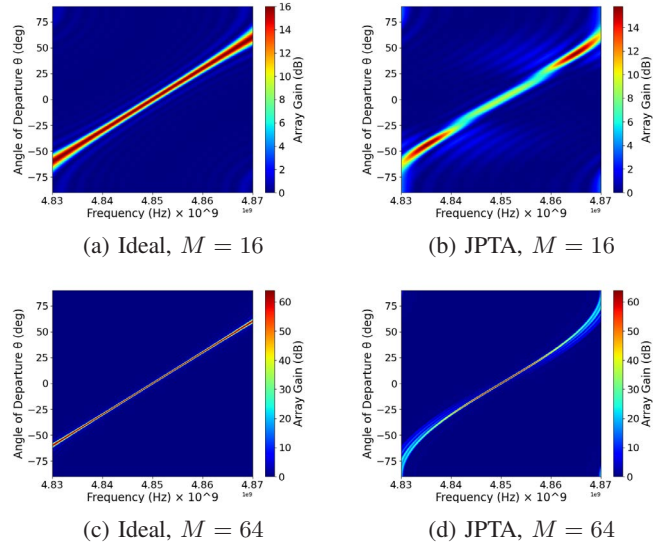


Fig. 4: Illustration of the BF gain: $|\mathbf{a}_k^H(\theta) \cdot \mathbf{TPd}_k|^2$, achievable with JPTA BF with algorithm in Section III to reproduce beam behaviors ($\theta_0 = 0, \Delta\theta = 2\pi/3$) with $I_{\text{max}} = 10$ and $\Delta\tau = 0.1\text{ns}$.

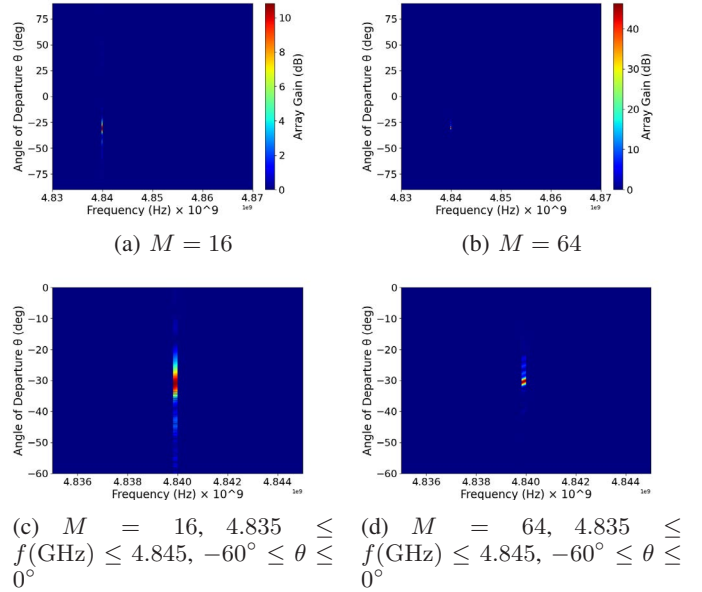


Fig. 5: Illustration of the BF gain: $\sum_{k \in \mathcal{K}_r} |\mathbf{a}_k^H(\theta) \cdot \mathbf{TPd}_k|^2$, achievable with JPTA BF with algorithm in section III to replicate beam behaviors ($\theta_0 = 0, \Delta\theta = 2\pi/3$) with $I_{\text{max}} = 10$ and $\Delta\tau = 0.1\text{ns}$.

A. Results of Optimization Algorithms

Fig. 4 shows the BF gain of \mathcal{B} and \mathcal{C} for the number of antenna elements $M = 16$ and $M = 64$. From Fig. 4, JPTA has an approximately one-to-one mapping between the BF gain at a given angle and a specific frequency. From Fig. 4d, the change in directivity is larger at the frequencies at both

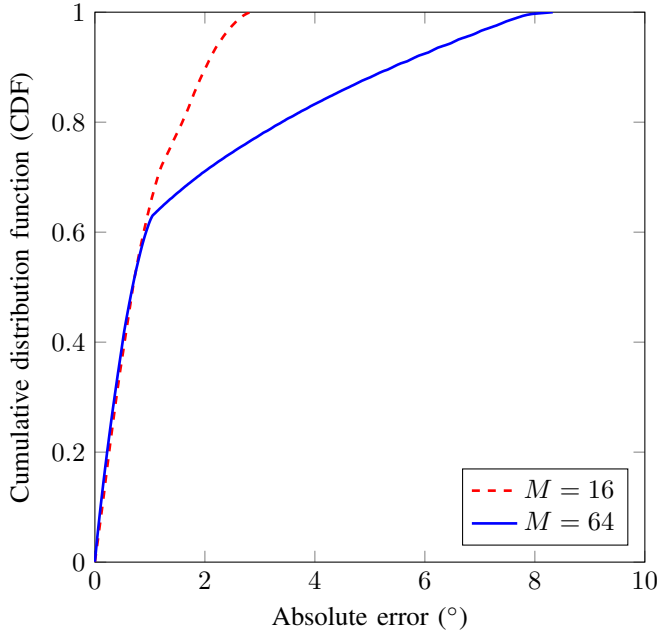


Fig. 6: The cumulative distribution function (CDF) of absolute error obtained by $|\theta_{\text{int}} - \theta^*|$.

ends of the bandwidth than in Fig. 4c. This may be due to the following reasons. From Eq. (3), the amount of phase required to direct the gain towards angle θ is given by

$$\pi \sin(\theta). \quad (15)$$

On the other hand, from Eq. (1), the amount of phase by the unit-norm analog JPTA BF is given by

$$\phi - 2\pi f_k \tau. \quad (16)$$

From Eq. (16), ϕ and τ are constants and Eq. (16) is a linear function. Therefore, the deviation from Eq. (15) appears as shown in Fig. 4d.

The JPTA BF gains of the 12 subcarriers in the RB $r = 54$ are shown in Fig. 5. From Fig. 5, we can see that the region of high received power at 1RB due to BF by the set \mathcal{C} of unit-norm analog JPTA BF corresponds to a specific angle. Therefore, the DoA estimation described in Section IV is possible using BF of \mathcal{C} .

B. Results of the proposed method

Next, the DoA estimation accuracy of the proposed method described in Section IV is evaluated. It is assumed that the interfering signal is coming from a randomly selected direction $\theta_{\text{int}} \in [-\pi/3, \pi/3]$. In this simulation, the signal $\mathbf{y}_k(\theta_{\text{int}})$ from the other system is assumed to be the unit-norm wave vector \mathbf{a}_k of Eq. (3). Therefore, Eq. (12) in the proposed method in Section IV is given by

$$r^* = \underset{r \in \mathcal{R}}{\operatorname{argmax}} \sum_{k \in \mathcal{K}_r} |\mathbf{a}_k^T(\theta_{\text{int}}) \cdot \mathbf{TPd}_k|^2. \quad (17)$$

Fig. 6 shows the cumulative distribution function (CDF) of the absolute error between θ_{int} and θ^* estimated using the proposed method in Section IV. The red line is the result for $M = 16$ and the blue line is the result for $M = 64$. From Fig. 6, it can be seen that it is possible to estimate the DoA with a maximum error of about 3° for $M = 16$ and a maximum error of about 8° for $M = 64$. The proposed method finds the frequency with the highest received power and the angle corresponding to the obtained frequency is derived. Thus, the derivation of the actual JPTA BF from the ideal case leads to a difference in DoA estimation. Focusing on $M = 64$, the accuracy of DoA estimation is good around $\theta_{\text{int}} = 0$. However, around $\theta_{\text{int}} \pm \pi/3$, the accuracy deteriorates due to derivation of the JPTA BF from the ideal case. For $M = 16$, the difference between Ideal and JPTA is not as large as for $M = 64$, but the number of antennas is smaller and the beamwidth is larger than for $M = 64$. Therefore, we consider that up to a certain percentage, the estimation accuracy is better for $M = 64$ than for $M = 16$, and beyond that percentage, the estimation accuracy for $M = 64$ is much worse in Fig. 6.

VI. CONCLUSION AND FUTURE WORK

This paper proposed the JPTA-based DoA estimation method to accurately estimate the DoA of the interfering signal while reducing the required overhead. The proposed method focused on the relationship between the frequency and its corresponding DoA with high BF gain. Computer simulation results have shown that the proposed method can estimate the DoA with the accuracy of 2° at a 90-percentile value.

One of the future works is to improve the accuracy of the DoA estimation. We are considering a specific method to improve the accuracy by mapping the error in the correspondence between the ideal beam behavior and the JPTA beam behavior in terms of frequency and DoA.

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