High-Speed Resource Allocation for Multi-User NOMA Systems Using a Coherent Ising Machine

Shuho Ishibashi*, Maki Arai*, Aohan Li†, Hiroki Takesue‡ , Kensuke Inaba‡, Kazuyuki Aihara§ and Mikio Hasegawa*
*Department of Electrical Engineering Graduate School of Engineering,
Tokyo University of Science, 6-3-1, Niijuku, Katsushika-ku, Tokyo 125-8585, Japan,
Email: 4323504@ed.tus.ac.jp, maki.arai@rs.tus.ac.jp, hasegawa@ee.kagu.tus.ac.jp
†Graduate School of Informatics and Engineering,
The University of Electro-Communications
1-5-1 Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan
Email: aohanli@ieee.org
‡NTT Basic Research Laboratories, NTT Corporation,
Atsugi, Kanagawa, 243-0198, Japan
Email: hiroki.takesue@ntt.com, kensuke.inaba@ntt.com
§International Research Center for Neurointelligence,
The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
Email: kaihara@g.ecc.u-tokyo.ac.jp

***Abstract*—Non-Orthogonal Multiple Access (NOMA) is a next-generation communication technology that enables multiple users share the same wireless channels. To optimize the performance of NOMA, appropriate channel allocation for each user is essential. Previous research has applied a fast optimization approach using the coherent Ising machine (CIM) to channel allocation in NOMA, multiplexing up to two users per channel, which can be formulated as a basic Ising Hamiltonian including up to second-order interactions. However, NOMA can support multiplexing more than two users per channel depending on channel conditions. This paper propose a method for optimizing NOMA systems with more than two multiplexed users per channel, which becomes a Higher Order Binary Optimization problem. We apply a method to reduce the order of the objective function, converting it to Quadratic Unconstrained Binary Optimization, and derive the Ising Hamiltonian for higher-order NOMA. We evaluate our proposed method through simulations using the CIM for optimization. Our results show that the proposed method leads to better solutions than conventional methods.**

***Index Terms*—Non-Orthogonal Multiple Access, Coherent Ising Machine, Resource Allocation, Quadratic Unconstrained Binary Optimization, Higher Order Binary Optimization.**

I. INTRODUCTION

In recent years, with the spread of various internet services, data traffic has been increasing significantly, necessitating the further expansion of bandwidth [1]. To address this issue, a technology called Non-Orthogonal Multiple Access (NOMA) has been proposed [2]. NOMA is a technique that multiplexes users in the power domain, allowing multiple users to be allocated on the same channel at the same time. In a NOMA system, the transmitter applies superposition coding to transmit the multiplexed signals, while the receiver utilizes Successive Interference Cancellation (SIC) to distinguish the signals and extract the desired one [3]. Consequently, NOMA offers advantages such as improved spectral efficiency, massive connectivity, and low transmission latency.

However, one of the challenges with NOMA is its heavy dependence on the combination of channels and the users assigned to them. To achieve high performance, it is essential to solve a combinatorial optimization problem that selects the optimal user-channel combinations in real time and at high speed.

Therefore, this study utilizes coherent Ising machine (CIM) [4] [5], which has been applied to various combinatorial optimization problems in wireless communication systems. D-Wave [6] is also known as a quantum annealer as a machine that solves combinatorial optimization problems at high speed. A prior study [7] showed that the CIM outperforms D-Wave in solving large-scale, high-density problems, highlighting its suitability for next-generation large-scale wireless communications. Additionally, studies [8]- [11] have utilized CIMs to optimize various wireless communication systems.

The application of CIM to NOMA system optimization has already been considered [14]. Previous studies have reported that the CIM can solve the optimization problem in a downlink NOMA system, where a maximum of two users are multiplexed on the same channel, within milliseconds while maintaining high solution accuracy. On the other hand, in environments with narrow areas, the benefits of NOMA can be further leveraged by increasing the number of users multiplexed on a single channel to three or more.

This paper addresses the optimization problem of resource allocation in a downlink NOMA system where up to three or more users are multiplexed on the same channel. The CIM rapidly solves combinatorial optimization problems rapidly by transforming them into minimizing the Ising Hamiltonian in an

Ising model. For this transformation, the objective function of the combinatorial optimization problem must match the degree of the Hamiltonian. While the conventional Hamiltonian is a quadratic expression, it is necessary to reduce the degree when dealing with combinatorial optimization problems with higher-order objective functions. This paper proposes a method to solve the resource allocation optimization problem for multi-user NOMA systems using the CIM by reducing the degree of the objective function. We evaluate the performance of the proposed method with applying to resource allocation in the NOMA system superimposing three users each channel.

II. Coherent Ising Machine

The CIM [12] is a hardware that reproduces Ising spins using optical pulses and rapidly searches for a spin configuration that corresponds to the ground state of the energy structure of the Ising model. The Ising model represents magnetic spins in one of two states: up or down. These spins interact with each other through mutual coupling and are also affected by external magnetic fields. The Ising Hamiltonian, which represents the energy structure of the Ising model, is shown below:

$$H_{\text{Ising}} = -\frac{1}{2}\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} J_{ij}\sigma_i\sigma_j + \sum_{i\in\mathcal{M}}\lambda_i\sigma_i. \tag{1}$$

where $\sigma_i \in \{-1, +1\}$ is the spin direction, J_{ij} is the strength of interaction, λ_i is the strength of external magnetic field. CIM can obtain the ground state of this Ising Hamiltonian. Hence, if we find $\boldsymbol{J}$ and $\boldsymbol{\lambda}$ for the optimization problem, we can quickly obtain the optimal solution.

Originally, the CIM was implemented as a laser network composed of a master laser and multiple slave lasers [12]. More recently, a measurement-feedback-based CIM has been developed, enabling an easier increase in the number of spin couplings to address larger-scale problems [13]. While D-Wave emulates spin coupling using a fixed topology with sparse connections, the CIM employs an Ising network of optical pulses with full coupling.

III. System Model and Problem Formulation

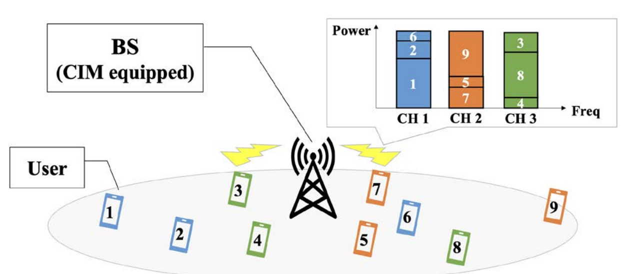

Fig. 1. System Model

Figure 1 illustrates the overview of the system model. This paper considers a downlink NOMA system with a circular cell structure where a single base station transmits data to multiple users. The sets of users and channels within the cell are denoted as $\mathcal{U} = \{1, ..., U\}$ and $\mathcal{C} = \{1, ..., C\}$, respectively. For simplicity, we assume that a maximum of three users can be multiplexed on the same channel, representing the simplest case of multiple users. In this study, since three users are multiplexed on the same channel, the data rate R^j_{ikm} for user $i \in \mathcal{U}$ assigned to channel $j \in \mathcal{C}$ along with users $k, m \in \mathcal{U}$ is expressed as follows:

$$R^j_{ikm} = \begin{cases} B_j \log_2\left(1 + P^j_{ikm}\Gamma^j_i\right), & \Gamma^j_i > \Gamma^j_k > \Gamma^j_m \\ B_j \log_2\left(1 + \frac{P^j_{ikm}\Gamma^j_i}{1+P^j_{kim}\Gamma^j_i}\right), & \Gamma^j_k > \Gamma^j_i > \Gamma^j_m \\ B_j \log_2\left(1 + \frac{P^j_{ikm}\Gamma^j_i}{1+P^j_{kim}\Gamma^j_i + P^j_{mik}\Gamma^j_i}\right), & \Gamma^j_k > \Gamma^j_m > \Gamma^j_i. \end{cases} \tag{2}$$

where Γ^j_i represents the carrier to noise ratio (CNR) of user i when assigned to channel j. Regarding power allocation, assuming that the CNR of users i, k, and m are in increasing order distance from the base station, it is defined as follows:

$$\begin{cases} \text{user } i: \ P^j_{ikm} = 0.01q_j \\ \text{user } k: \ P^j_{kim} = 0.09q_j \\ \text{user } m: \ P^j_{mik} = 0.90q_j \end{cases} \tag{3}$$

where q_j represents the power allocated to channel j. The weights are set to increase approximately tenfold as the distance from the base station increases, ensuring that the receiver's SIC can effectively cancel out other signals.

In this paper, the total data rate of the system is used as the optimization metric, and the objective function and constraints are defined as follows:

$$\max_{x} \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{\substack{k\in\mathcal{U}\\k\neq i}}\sum_{\substack{m\in\mathcal{U}\\m\neq i,k}} (R^j_{ikm} + R^j_{kim} + R^j_{mik})x_{ij}x_{kj}x_{mj}. \tag{4}$$

$$s.t. \quad \sum_{j\in\mathcal{C}} x_{ij} = 1, \quad \text{for } \forall i. \tag{5}$$

$$\sum_{i\in\mathcal{U}} x_{ij} = 3, \quad \text{for } \forall j. \tag{6}$$

$$x_{ij} = \begin{cases} 1, & \text{if the } i\text{th user uses the } j\text{th channel.} \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

where this binary variable x_{ij} corresponds to the spin orientation σ_{ij} in the CIM. Eq.(4) represents the objective function, while Eq.(5) and Eq.(6) represent the constraints. Eq.(5) is a constraint ensuring that a user is assigned to only one channel, and Eq.(6) limits the number of users assigned to a single channel to a maximum of three.

IV. Proposed Method to Optimize 3-User NOMA using CIM

To solve optimization problems using the CIM, it is necessary to formulate it as the Ising Hamiltonian and derive $\boldsymbol{J}$ and $\boldsymbol{\lambda}$. In the optimization problem of this study, four spin indices are required, and thus it is formulated in the form of the Ising Hamiltonian as shown in Eq.(8):

$$H_{\text{Ising}} = -\frac{1}{2}\sum_{\substack{i,m\in\mathcal{M},\\ j,n\in\mathcal{N},\\ k,p\in\mathcal{K},\\ l,q\in\mathcal{L}}} J_{ijklmnpq}\sigma_{ijkl}\sigma_{mnpq} + \sum_{\substack{i\in\mathcal{M},\\ j\in\mathcal{N},\\ k\in\mathcal{K},\\ l\in\mathcal{L}}} \lambda_{ijkl}\sigma_{ijkl} \tag{8}$$

Based on Eqs.(4), (5), and (6), the optimal solution for the resource allocation problem can be obtained by minimizing the following function:

$$E_{\text{NOMA}} = AE_1 + BE_2 + DE_3. \tag{9}$$

$$E_1 = -\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{\substack{k\in\mathcal{U}\\ k\neq i}}\sum_{\substack{m\in\mathcal{U}\\ m\neq i,k}}(R^j_{ikm} + R^j_{kim} + R^j_{mik})x_{ij}x_{kj}x_{mj}. \tag{10}$$

$$E_2 = \sum_{i\in\mathcal{U}}\left(\sum_{j\in\mathcal{C}}x_{ij} - 1\right)^2. \tag{11}$$

$$E_3 = \sum_{j\in\mathcal{C}}\left(\sum_{i\in\mathcal{U}}x_{ij} - 3\right)^2. \tag{12}$$

where A, B and D are scaling parameters. Since Eq.(4) is a cubic function of $\boldsymbol{x}$, whereas the Ising Hamiltonian is a quadratic function of $\boldsymbol{\sigma}$, it is necessary to reduce the degree of Eq.(4). Therefore, a new variable $\boldsymbol{y}$ [15] is introduced as follows:

$$y_{ijkl} = x_{ij}x_{kl}. \tag{13}$$

By substituting $\boldsymbol{x}$ with $\boldsymbol{y}$ in Eq.(10), the following form, which is a quadratic function, can be obtained:

$$E_1 = -\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{\substack{k\in\mathcal{U}\\ k\neq i}}\sum_{\substack{m\in\mathcal{U}\\ m\neq i,k}}\{(R^j_{ikm}+R^j_{kim}+R^j_{mik})y_{ijkj}y_{mjmj} - H_{\text{C}}\}. \tag{14}$$

where H_{C} is a constraint term to satisfy Eq.(13). In this paper, the following polynomial is used as H_{C}:

$$\begin{aligned} H_{\text{C}} &= \alpha\{x_{ij}x_{kj} - 2(x_{ij} + x_{kj})y_{ijkj} + 3y_{ijkj}\} \\ &= \alpha\{y_{ijij}y_{kjkj} - 2(y_{ijij} + y_{kjkj})y_{ijkj} + 3y_{ijkj}\}. \end{aligned} \tag{15}$$

where α is a scaling parameter. If Eq.(13) is satisfied, $H_{\text{C}} = 0$; if Eq.(13) is not satisfied, $H_{\text{C}} > 0$. Eq.(14) becomes a quadratic function of $\boldsymbol{y}$, which can be transformed into the form of Eq.(8).

First, we derive interactions and external magnetic fields for the variables $\boldsymbol{x}$ or $\boldsymbol{y}$, which take values of $\{0, 1\}$. The result of transforming Eq.(14) is as follows:

$$E_1 = -\frac{1}{2}\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{k\in\mathcal{U}}\sum_{l\in\mathcal{C}}\sum_{m\in\mathcal{U}}\sum_{n\in\mathcal{C}}\sum_{p\in\mathcal{U}}\sum_{q\in\mathcal{C}}W^1_{ijklmnpq}y_{ijkl}y_{mnpq} + \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{k\in\mathcal{U}}\sum_{l\in\mathcal{C}}\theta^1_{ijkl}y_{ijkl}. \tag{16}$$

$$W^1_{ijklmnpq} = \frac{1}{24}\delta_{jl}\delta_{ln}\delta_{nq}\delta_{qj}(1-\delta_{ik})(1-\delta_{km})(1-\delta_{im})\sum_{s=1}^{24}w(\pi_s). \tag{17}$$

$$w(\pi_s) = \left(\frac{1}{6}\sum_{t=1}^{6}\sum_{v\in\{j,l,n\}}R^v_{(\xi_t)}\right)\delta_{im}\delta_{jn} - \alpha\delta_{ik}\delta_{jl}\delta_{mp}\delta_{nq} + 2\alpha\delta_{im}\delta_{jn}\delta_{ip}\delta_{jq} + 2\alpha\delta_{mk}\delta_{nl}\delta_{pk}\delta_{ql}. \tag{18}$$

$$\theta^1_{ijkl} = -\frac{3}{2}\alpha\delta_{jl}\delta_{ln}\delta_{nq}\delta_{qj}(1-\delta_{ik})(1-\delta_{km})(1-\delta_{im}). \tag{19}$$

where δ_{ij} is the Kronecker's delta: δ_{ij}=1 if i=j and δ_{ij}= 0 otherwise. π_p and ξ_s represent permutations of the sets $\{(i,j),(k,l),(m,n),(p,q)\}$ and $\{i,k,m\}$, respectively. In other words,

$$\begin{aligned} \pi_1 &= \{(i,j),(k,l),(m,n),(p,q)\}, \\ \pi_2 &= \{(i,j),(k,l),(p,q),(m,n)\}, \\ &..., \\ \pi_{24} &= \{(m,n),(p,q),(k,l),(i,j)\}, \\ \xi_1 &= \{i,k,m\}, \\ \xi_2 &= \{i,m,k\}, \\ &..., \\ \xi_6 &= \{m,k,i\}. \end{aligned}$$

Eqs.(11),(12) can also be rewritten in terms of $\boldsymbol{y}$ as follows:

$$\begin{aligned} E_2 &= \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{k\in\mathcal{U}}\sum_{l\in\mathcal{C}}\left(\delta_{ik} - \frac{1}{2}\delta_{ik}\delta_{jl}\right)x_{ij}x_{kl} \\ &= \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{k\in\mathcal{U}}\sum_{l\in\mathcal{C}}\theta^2_{ijkl}y_{ijkl}. \end{aligned} \tag{20}$$

$$\begin{aligned} E_3 &= \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{k\in\mathcal{U}}\sum_{l\in\mathcal{C}}\left(\delta_{jl} + \frac{3}{2}\delta_{ik}\delta_{jl}\right)x_{ij}x_{kl} \\ &= \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{C}}\sum_{k\in\mathcal{U}}\sum_{l\in\mathcal{C}}\theta^3_{ijkl}y_{ijkl}. \end{aligned} \tag{21}$$

From Eqs.(16),(17),(20),(21), interactions and external magnetic fields for the variables $\boldsymbol{x}$ or $\boldsymbol{y}$ can be obtained:

$$\begin{aligned} W^{\text{NOMA}}_{ijklmnpq} &= W^1_{ijklmnpq}, \\ \theta^{\text{NOMA}}_{ijkl} &= \theta^1_{ijkl} + \theta^2_{ijkl} + \theta^3_{ijkl}. \end{aligned} \tag{22}$$

Finally, we derive interactions and external magnetic fields for the variable $\boldsymbol{\sigma}$, which take values of $\{-1, +1\}$. By using $\sigma_{ijkl} = 2y_{ijkl} - 1$ to convert the variable $\boldsymbol{y}$ into the spin orientation $\boldsymbol{\sigma}$, $\boldsymbol{J}$ and $\boldsymbol{\lambda}$ are obtained as follows:

$$\begin{aligned} J^{\text{NOMA}}_{ijklmnpq} &= \frac{W^{\text{NOMA}}_{ijklmnpq}}{2}, \\ \lambda^{\text{NOMA}}_{ijkl} &= \theta^{\text{NOMA}}_{ijkl} - \sum_{m\in\mathcal{U}}\sum_{n\in\mathcal{C}}\sum_{p\in\mathcal{U}}\sum_{q\in\mathcal{C}}\frac{W^{\text{NOMA}}_{ijklmnpq}}{2}. \end{aligned} \tag{23}$$

By setting the derived $\boldsymbol{J}$ and $\boldsymbol{\lambda}$ to CIM, the Ising spin state corresponding to the optimal solution can be rapidly obtained and by checking the status of each spin, the user allocation of 3-user NOMA can be obtained.

V. Results

TABLE I
PARAMETERS

Parameter	Value
Cell radius	200, 300, 400, 500 m
Minimum distance between user and BS	50 m
Total bandwidth	5.0 MHz
Path loss coefficient	3, 4

In this section, the performance of the proposed method is evaluated through simulations. Table 1 shows the simulation parameter settings. We compare the proposed method with Exhaustive Search (ES), Random allocation, and Simulated Annealing (SA). ES obtains the optimal solution by exhaustively evaluating all possible user assignments. Random Allocation assigns users randomly. SA replicates the annealing process in metallurgy on a computer and is known for being less likely to get trapped in local optima due to its temperature parameter.

Fig. 2 illustrates a comparison of total data rates when the number of users is varied. Fig. 3 compares the total data rate in a line-of-sight communication environment. To account for cases where the number of users is not a multiple of 3, we define the dummy users and adjust the total number of users to a multiple of 3. In Fig. 2 and 3, the number of channels is set to 3 for each case.

From Fig. 2 and 3, it can be confirmed that the proposed method achieves a solution close to the optimal. Furthermore, since the CIM can obtain solutions at a speed on the scale of milliseconds, it can solve problems much faster than other methods.

Fig. 4 illustrates a comparison of total data rates when the cell radius is varied. From Fig.4, since throughput is higher when the cell radius is smaller, it can be said that multi-user NOMA communication is more effective for short-distance communication.

VI. Conclusion

In this paper, we solved multi-user NOMA optimization problems efficiently by applying CIM. Since the multi-user NOMA optimization problem results in a cubic objective function, we propose a method that applies order reduction. The simulation results confirmed that the proposed method achieved a total data rate close to the optimal solution compared to other methods.

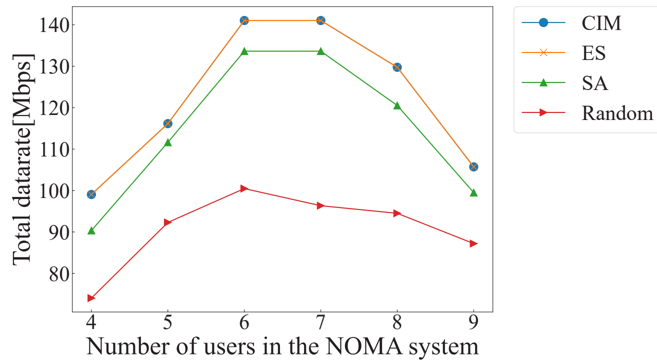

Fig. 2. Total datarate for a varying number of users (Cell radius = 500m, Pathloss coefficient = 4).

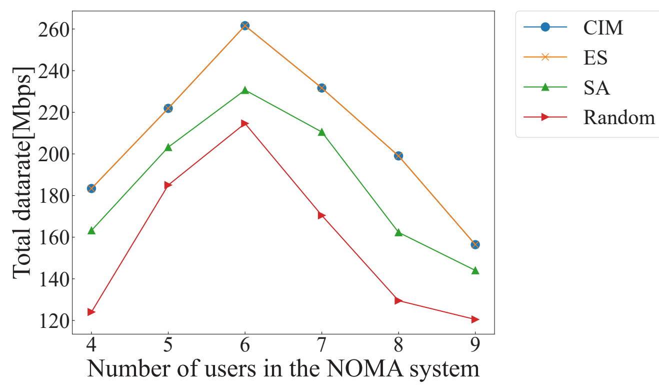

Fig. 3. Total datarate for a varying number of users (Cell radius = 200m, Pathloss coefficient = 3).

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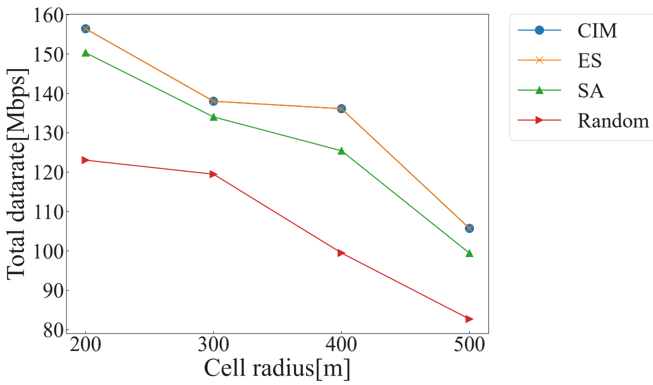

Fig. 4. Total datarate for a varying cell radius ($U = 9$, $C = 3$, Pathloss coefficient = 3).

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