Node Criticality Assessment in Interdependent Networks via Decoupled Function Graphs

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***Abstract*—The traditional topology-based metrics for assessing node criticality may fall short in highly heterogeneous multilayer networks. This work introduces a novel modeling framework that incorporates both function and topology to effectively measure node criticality. The framework is applied to the State of Kansas' transportation network as a practical example of an infrastructure network, identifying the most critical nodes under various operating load conditions. The findings reveal, among other insights, a significant rise in an adversary's success rate when targeting the most critical nodes rather than a random set of nodes.**

***Index Terms*—Self-Healing Cyber-Physical Systems, Transportation Networks, Cascading Failure, Node Criticality.**

I. Introduction

Today's interconnected infrastructure networks are highly heterogeneous and complex systems that rely on advanced technology and integration across various sectors. Therefore, when studying network resilience, assessing the criticality of a node cannot be based solely on its topology; the node's function must also be considered. However, the sheer complexity of modern infrastructure networks, with their numerous components and subsystems, makes this task challenging. In particular, creating accurate models that capture the behavior of interconnected networks under stress is difficult. Additionally, large-scale simulations of such networks can be computationally intensive and require significant resources. Thus, it is crucial to introduce simple models of real-life networks and develop scalable frameworks to study them.

With this in mind, this work aims to address the following questions:

- How can we assess the most critical nodes in a highly heterogeneous network?
- How can we devise a flexible framework that captures topology, function, and temporality?

There is a vast body of literature on node criticality (see, for example, [1]–[13]). However, most existing work either limits the criticality analysis solely to the topological aspect or does not adequately capture node function in assessing criticality. We propose a framework that utilizes decoupled function graphs for studying interdependent networks. This framework is scalable to large networks and can be applied to multiple layers of interconnected networks, including structures where one network can help recover the damaged nodes in another. As a proof of concept, we apply this framework to a simple model of a real-life interconnected transportation network.

The rest of this paper is organized as follows. Section II outlines the proposed framework. Section III describes the models of the network, contagion, and healing (node recovery), and defines a metric for resilience. Section IV provides the numerical results and insights derived from them. Finally, Section V concludes the paper.

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II. Proposed Framework

We propose to represent each network function as a homogeneous "function graph" and then use graphical models to overlay multiple layers of such graphs. This approach results in a multi-layer graph, which is well-suited for applying powerful inference algorithms such as message passing to analyze the dynamics and steady state of the entire model.

To illustrate the proposed "network decomposition," consider the example in Fig. 1(a), which presents a toy network of 10 nodes. The interaction between two nodes can be of two types: Type A and/or Type B. A Type A interaction is represented by a blue link between two nodes, while a Type B interaction is shown by a red link. A pair of nodes in the network can be connected through one or both types of links.

In Fig. 1(b), we show a "decomposed" equivalent of the toy network into two interconnected homogeneous function graphs. The overlay network (square nodes) captures only the Type A links, whereas the underlay network (circle nodes) captures only the Type B interactions. For any node in the toy network of Fig. 1(a), there is a corresponding node in the overlay network and one in the underlay network in Fig. 1(b), connected by a dashed line. The functionality of each node is modeled mathematically as $\underline{o} = f(\underline{i})$, where $\underline{o}$ and $\underline{i}$ are the output and input vectors, respectively.

III. Models, Methods, and Metrics

The proposed framework is applied to the transportation network of my home state of Kansas, which includes a combination of state and interstate highways, as well as local and international airports. We begin by developing a model of this network and then derive the corresponding layers of function graphs for both the highway and air traffic networks.

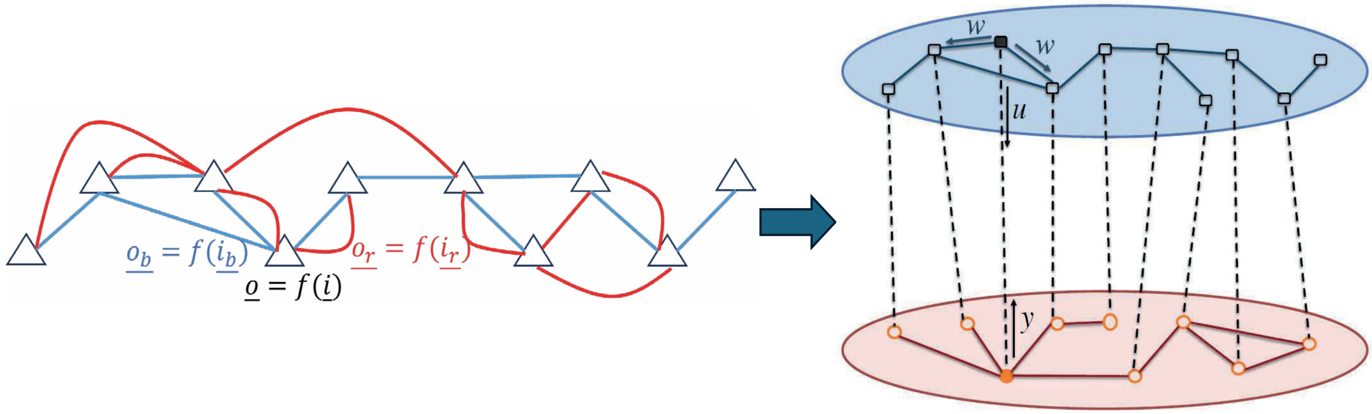

Fig. 1: Example of decomposition of a network into two function graphs: (a) a toy network with two types of links, blue and red, (b) the equivalent decomposed version of the toy network into two interconnected homogonous layers.

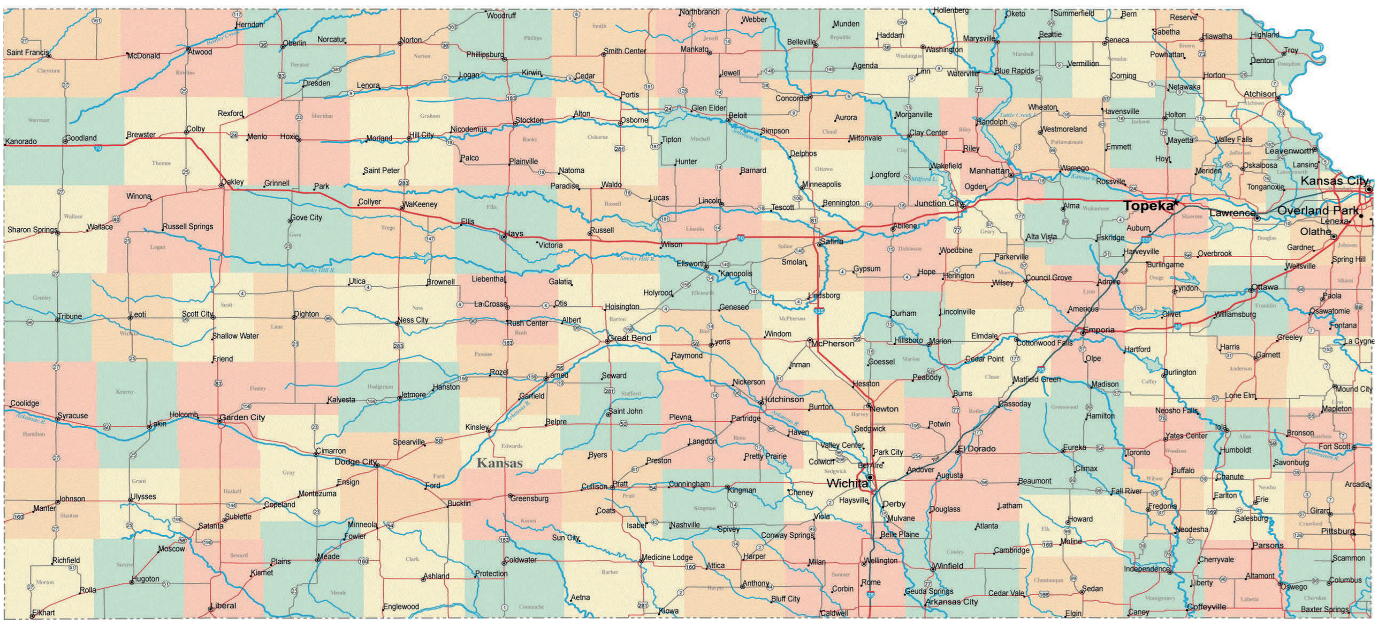

Fig. 2: The Kansas road map shows the state's major roads and highways as well as its counties and cities [14].

Traffic data for each network was collected and utilized to derive models of the operational functions for each node type.

A. Modeling Kansas' Road Network

Fig. 2 depicts the Kansas road map [14]. To simplify the presentation of results, we considered only 19 cities and towns with populations of approximately 22,000 or more. The resulting simplified road network connecting these major cities is shown in Fig. 3, which includes 46 edges. Each edge is assigned a unique ID number from 0 to 45 to facilitate the explanation of numerical results later.

It's important to note that in Fig. 2, any two cities, X and Y, might be connected through multiple routes. However, for their nodes to be connected by an edge in the graph of Fig. 3, the route connecting them must not have been used to connect X or Y to another city, Z. In other words, we are interested in the network's resiliency and only connect two cities if an independent route exists between them that would remain viable if other routes fail. Therefore, "reachability" is our primary concern.

We assume that, in the absence of disturbances, each road operates under a normal load that is less than its capacity. Formally, a road can handle up to $(1 + \alpha)$ times its normal load, where $\alpha > 0$ is referred to as the "overhead parameter." When a road reaches this maximum allowed traffic load, it is considered to be operating at its "capacity." If a road is subjected to a load higher than its capacity, it will continue operating at its capacity, distributing the excess load to its neighboring roads, which are represented in Fig. 3 by edges adjacent to both ends of the road. For instance, the neighboring roads to road 1 include roads 0, 3, 2, 5, 4, and 11.

We assume that an interstate or toll road has twice the capac-

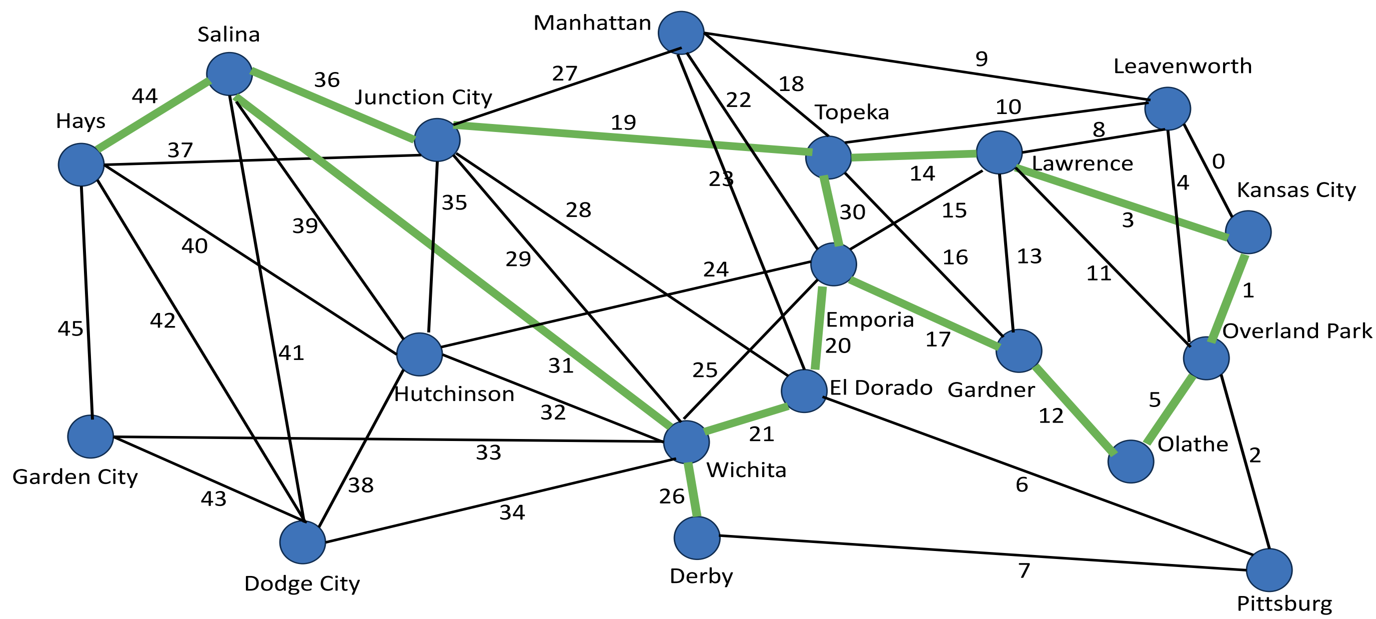

Fig. 3: A simplified network model of the Kansas road map connecting the major centers of population. Each road is assigned an ID. Interstate and toll roads are assumed to have twice the capacity of other roads and are shown by thick green edges.

ity of a US highway, state road, or other road were chosen for simplicity; any two numbe relative traffic loads of the routes would suffic interstate and toll roads are represented in Fig. edges, while other roads are depicted by thir

It is important to note that, for the purpo we did not consider the length of the routes. the edges between the nodes in Fig. 3 do not attribute. Extending the proposed framework length and other attributes would be an intere future research.

B. Modeling Kansas Air Travel Network

Kansas is served by two major airports, ' Kansas City MCI. In addition, there are five m as "commercial" by FAA that can serve small planes [15]. Overall, these seven airports are For simplicity, we assumed that the air traf complete graph where travel between all air if the airports are operational.

C. Coupled Kansas Air-Ground Transportation Network

By overlaying the air traffic network of Fig. 4 on the road network of Fig. 3, we obtain a coupled air-ground transportation network for Kansas, as presented in Fig. 5. In this network, each airport, except Kansas City MCI, is connected to its corresponding city. MCI, situated between Kansas City and Leavenworth, serves both cities.

D. Initial Disturbance, Contagion, and Resilience

The resilience of a transport system refers to its ability to resume operations at a level similar to that before a disruption occurred. The less disruption in terms of capacity and fluidity,

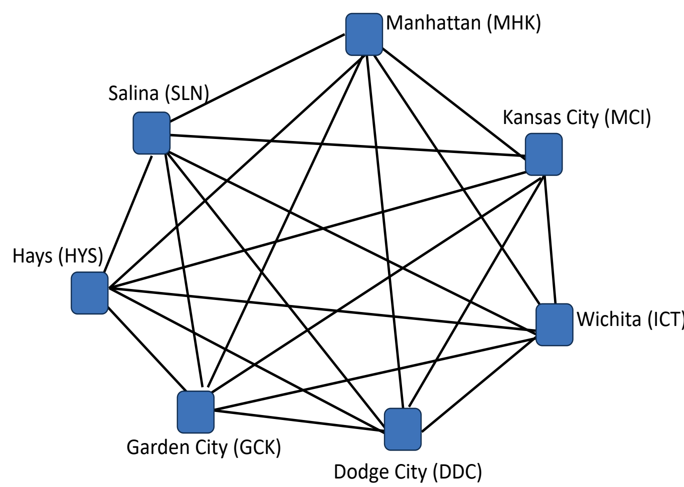

Fig. 4: A simplified model of the Kansas air traffic network.

and the quicker a system resumes normal operations, the higher its resilience. Here, we outline our models for initial disturbance, contagion within the networks, healing, and the resilience metric. Our methodology can be extended to a wide range of models.

1) Initial Attack and the Resulting Contagion:

- **Focus on Road Network Damage:** We assume the adversary has the resources to damage 5 out of the 46 roads shown in Fig. 2.
- **Traffic Redistribution:** When a road is hit, its traffic load must be picked up by its neighboring roads, with the load distributed equally among them.
- **Road Stress:** If a road reaches its capacity, it is considered to be under "stress." In this state, any unwanted

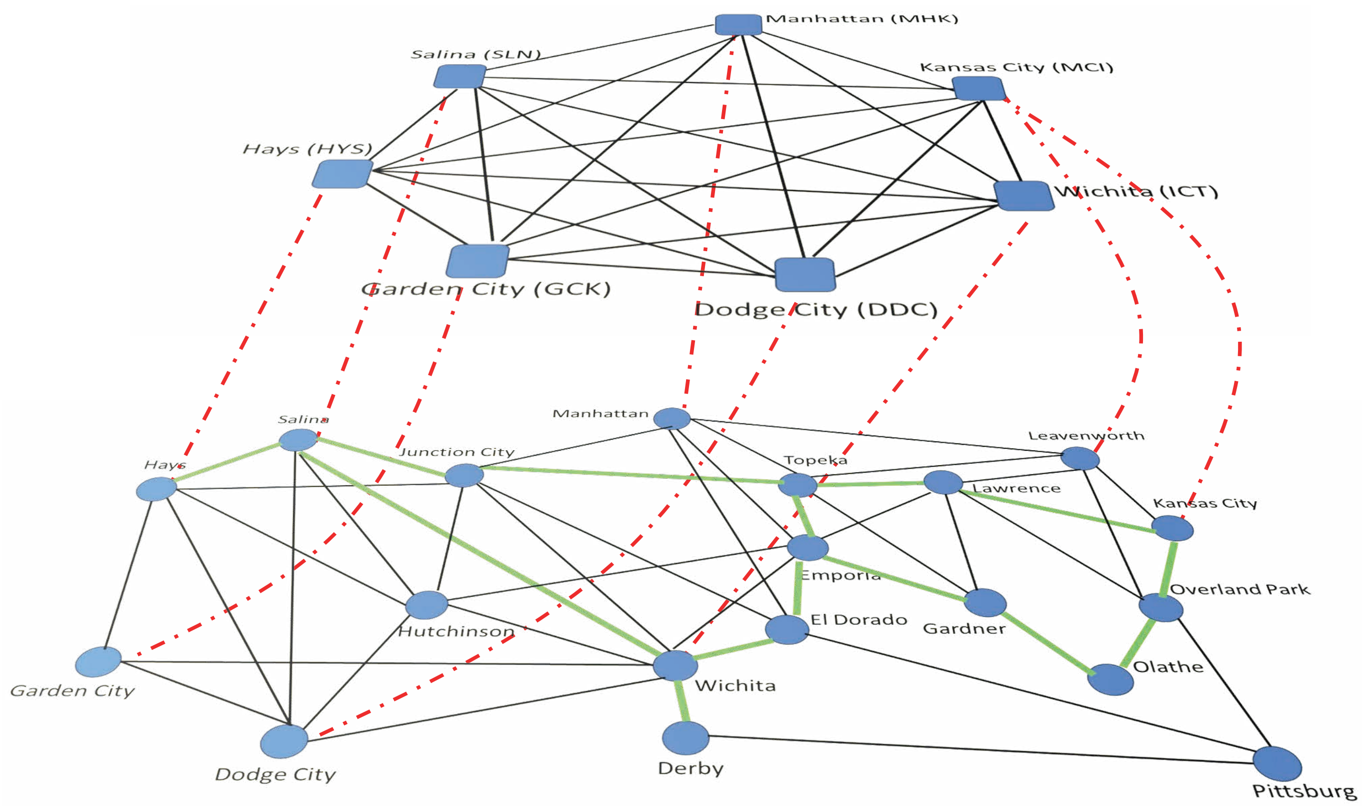

Fig. 5: A model of the coupled Kansas air-ground transportation network.

event, such as an accident, will cause the road to go out of service.

- **Overloaded Roads:** If a road becomes overloaded due to the burden of the load from a damaged neighboring road, it will continue operating at its capacity, distributing the extra load to its neighbors.
- **Contagion Limitation:** This study does not consider contagion of failures from the road network to the air network, nor does it consider contagion within the air network.

2) Healing:

- **Role of Airports:** Airports are used to heal the road network by bringing in critical units and supplies to sustain a city and repair its damaged adjacent roads, if any.
- **Road Healing Time:** If a damaged road is adjacent to a city served by an airport, it is considered to be "served" by an airport. A damaged road served by an airport will heal in 5 time units; otherwise, it will heal in 10 time units. A unit of time is defined as the time taken for load redistribution between a road and its neighbors. These numbers (5 and 10) are chosen for simulation perspective and do not alter the intuition provided by the analysis and results.

3) Resilience: We consider two factors for network resilience:

- **Recovery Time:** The time needed for the network to fully recover.
- **Stress Size:** The extent of stress in the network, defined as the total number of damaged and stressed roads due to the attack.

The adversary's goal is to select five target roads to maximize stress size and recovery time. Hence, the adversary's success metric for a particular attack is the product of stress size and recovery time. The minimum value of this product would theoretically be $5 \times 5 = 25$ (if only the initial 5 nodes are damaged and it takes only 5 time units to fix them). We define the adversary's success metric S as:

$$S = \frac{\text{stress size} \times \text{recovery time}}{25}$$

The adversary aims to maximize S. We define the network's resilience R as:

$$R = \frac{1}{\max(S)}$$

Since $S \geq 1$, R will always be less than or equal to 1.

IV. Numerical Results

We applied the proposed framework to the model of Fig. 5, under the disturbance, contagion, and healing model explained in the previous section. Fig. 6 shows the numerical results obtained for various values of the overhead parameter, namely α = 0.5, 1, 1.5, and 2. For each value, we presented the following:

Overhead Parameter, α	Highest-Value Targets	Max Recovery Time	Max Stress Set	Max Success Metric	Random Attack's Success Metric	Network Resilience
0.5	{1, 3, 12, 17, 30}	10	{1, 3, 12, 17, 30, 0, 5, 11, 13, 15, 16, 4, 8, 14, 22, 24, 25, 2, 10}	7.6	2.40	0.13
1	{1,3,5,12,14}	10	{1,3,5,12,14,2,4,11,13,8,15,16}	4.8	1.93	0.2
1.5	{1, 3, 4, 5, 10}	10	{1, 3, 4, 5, 10, 0, 2, 11}	3.2	1.91	0.31
2	{0, 1, 2, 3, 5}	10	{0, 1, 2, 3, 5, 4, 11}	2.8	1.91	0.36

Fig. 6: Numerical results of applying the developed framework to the Kansas transport network.

(a) **Highest value targets:** The initial set of 5 damaged roads yielding the maximum success metric for the adversary, i.e., the highest value targets.
(b) **Max recovery time:** The recovery time resulting from the initial damage in (a).
(c) **Max stress set:** The set of damaged and stressed roads resulting from the attack in (a).
(d) **Max success metric:** The success metric obtained from the attack in (a), representing the maximum possible success metric achieved by the adversary.
(e) **Random attack's success metric:** The success metric achieved by the adversary if, instead of (a), a random set of 5 roads was attacked.
(f) **Network resilience:** Network resilience as defined in the previous section.

Several observations can be made from the results shown in Fig. 6, among which the following are particularly interesting:

- Roads 1, 3, and 5 are the most common in all cases, distinguishing them as the highest value targets.
- The difference in the adversary's success metric between a carefully chosen target set and a randomly chosen one is significant. This difference grows larger as α becomes smaller, i.e., as roads operate closer to their capacity. This highlights the importance and impact of strategic target selection.

V. Conclusion

We employed the concept of function graphs to model interdependent heterogeneous networks, using this approach to study network resilience. Specifically, we developed a functional model of Kansas' transportation infrastructure. Our method and metric allow us to rank nodes by their criticality, identify the highest-value assets within the network, and measure an adversary's success in targeting different sets of nodes. For future work, it would be intriguing to integrate temporality into the proposed framework.

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