Performance Analysis of Doppler Shift Positioning with Different Frequency Bands

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Abstract

The low signal strength of Global navigation satellite systems (GNSS) hinders accurate positioning in urban and indoor environments, as GNSS operates at high altitudes and uses medium Earth orbit (MEO) constellations. Low Earth orbit (LEO) satellite constellations are an alternative in such scenarios, as LEO satellites have stronger signals strength than GNSS. However, LEO satellites are not for navigation purposes, and the Signals of Opportunity (SOPs) approach is a popular way to use them for positioning. The Doppler shift is better suited to SOPs-based positioning. Moreover, the selection of the carrier frequency affects the positioning performance of the user terminal (UT). In this study, we compare the positioning outcomes of Doppler shift-based measurements with different carrier frequency bands. The comparison results confirm that higher carrier frequency results in better performance for Doppler positioning.

Ⅰ. Introduction

In the outdoor positioning system context, Global navigation satellite systems (GNSS) are the dominating positioning system, though performance degrades in urban and indoor environments due to low signal strength [1]. On the other hand, the low Earth orbit (LEO) satellites provide higher signal strength and resilience to jamming and spoofing, and are not designed for navigation purposes. The signals of opportunity (SOPs) approach opens the era of using communication signals for navigation purposes. Among the navigation observables, the Doppler shift can be easily extracted by differencing the transmitted and received frequencies.

Doppler shift positioning with GNSS is not popular due to poor positioning accuracy. However, the higher angular motion and low altitude of LEO satellites result in better positioning accuracy than the Doppler shift from GNSS [2]. The positioning performance also depends on the Doppler dilution of precision (DDOP), which is a metric that quantifies satellites' velocity directions and geometric diversity. The large constellation of LEO satellites enables the user terminal (UT) to select visible satellites with good DDOP, resulting in better positioning accuracy. Moreover, Doppler measurements are affected by atmospheric and environmental factors, which degrade positioning performance.

The Doppler shift measurement error is scaled by the transmit signal wavelength (i.e., transmit frequency) and translated into the positioning error of UT. Fig. 1 shows the system model for Doppler shift-based positioning. In this paper, we focus on evaluating the positioning performance of Doppler-based positioning with different transmit frequency bands from LEO satellites.

Ⅱ. Doppler Shift Positioning

UT extracts navigation observables, such as the Doppler shift from the communication signals of LEO satellites, which is known as the SOPs approzach. The Doppler shift measurement ($f_{\mathrm{d}}^{[\ell]}$) for the ℓ-th LEO satellite is a function of satellite and UT position as follows [1].

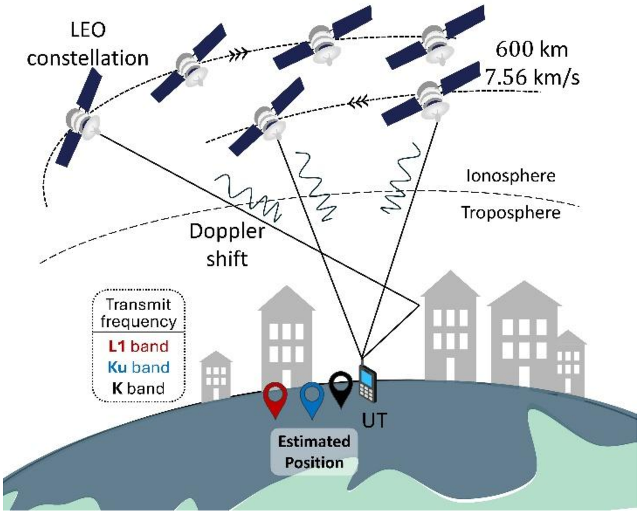

Figure 1. Doppler shift-based positioning with LEO satellites

$$f_{\mathrm{d}}^{[\ell]}\lambda^{[\ell]} = \mathbf{v}_{\mathrm{sat}}^{[\ell]} \cdot \frac{\mathbf{x}_{\mathrm{sat}}^{[\ell]}-\mathbf{x}_{\mathrm{ut}}}{\left\|\mathbf{x}_{\mathrm{sat}}^{[\ell]}-\mathbf{x}_{\mathrm{ut}}\right\|} + c\delta\dot{t}_{\mathrm{sat}} + c\delta\dot{t}_{\mathrm{UT}} + \epsilon, \qquad (1)$$

where $\mathbf{x}_{\mathrm{sat}}^{[\ell]}$, $\mathbf{v}_{\mathrm{sat}}^{[\ell]}$, $\lambda^{[\ell]}$, and $\mathbf{x}_{\mathrm{ut}}$ are the position, velocity, and transmit signal wavelength of the ℓ-th LEO satellite, and UT position, respectively. $\delta\dot{t}_{\mathrm{sat}}$ and $\delta\dot{t}_{\mathrm{UT}}$ are the clock drift effects of satellite and UT, respectively. ϵ is the error due to atmospheric effect, multipath effect, and measurement noise.

The relationship between the Doppler shift and UT position is nonlinear. The linearized equation with first-order Taylor approximation on an initial guess position ($\mathbf{x}_{0,\mathrm{ut}}$) is as follows [1].

$$f_{\mathrm{d}}^{[\ell]}\lambda^{[\ell]} \approx f_{0,\mathrm{d}}^{[\ell]}\lambda^{[\ell]} + \left[\left(\nabla f_{0,\mathrm{d}}^{[\ell]}\lambda^{[\ell]}\right)^{T} \quad \mathrm{c}\right]\begin{bmatrix}\Delta\mathbf{x}_{\mathrm{UT}}\\ \Delta\dot{t}\end{bmatrix} + \epsilon, \qquad (2)$$

where $f_{0,\mathrm{d}}^{[\ell]}$, $\nabla f_{\mathrm{d}}^{[\ell]}$, and $\Delta\mathbf{x}_{\mathrm{UT}}$ are the estimated Doppler shift, derivative of Doppler shift on initial estimation, and position error, respectively. With measurements from multiple satellites, (2) is written in matrix form as follows.

$$\Delta \mathbf{f}_{\mathrm{d}} \lambda = \mathbf{G} \Delta \mathbf{x} + \epsilon, \quad (3)$$

where $\Delta \mathbf{f}_{\mathrm{d}} \lambda = \mathbf{f}_{\mathrm{d}} \lambda - \mathbf{f}_{0,\mathrm{d}} \lambda$ and $\Delta \mathbf{x}^T = [(\Delta \mathbf{x}_{\mathrm{UT}})^T \quad \Delta t]$. $\mathbf{G}$ is the geometry matrix for Doppler shift positioning, and it is as follows:

$$\mathbf{G} = \begin{bmatrix} \left(\nabla f_{0,\mathrm{d}}^{[1]} \lambda^{[1]}\right)^T & \mathrm{c} \\ \vdots & \vdots \\ \left(\nabla f_{0,\mathrm{d}}^{[\ell]} \lambda^{[\ell]}\right)^T & \mathrm{c} \end{bmatrix} \quad (4)$$

The least-squares solution of (3) is

$$\Delta \mathbf{x} = \lambda (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \Delta \mathbf{f}_{\mathrm{d}}. \quad (5)$$

The estimation of optimal position involves iteratively adding the corrections to the initial estimation as

$$\mathbf{x}_{\mathrm{UT},k} = \mathbf{x}_{\mathrm{UT},k-1} + \Delta \mathbf{x}_k, \quad (6)$$

where k is the iteration number.

The position performance of UT depends on the DDOP and is as follows [3].

$$\mathrm{DDOP} = \sqrt{\mathrm{trace}[(\mathbf{G}^T \mathbf{G})^{-1}]}. \quad (7)$$

III. Positioning Error by Different Frequency Bands

The geometry matrix $\mathbf{G}$ is obtained by taking the derivative of the Doppler shift measurement ($f_{0,\mathrm{d}}^{[\ell]} \lambda^{[\ell]}$), which is obtained as [2].

$$\nabla f_{0,\mathrm{d}}^{[\ell]} \lambda^{[\ell]} = \frac{f_{0,\mathrm{d}}^{[\ell]} \lambda^{[\ell]} \mathbf{e}_{0,\mathrm{UT}}^{[\ell]} + \mathbf{v}_{\mathrm{sat}}^{[\ell]}}{\left\| \mathbf{x}_{\mathrm{sat}}^{[\ell]} - \mathbf{x}_{0,\mathrm{ut}} \right\|}. \quad (8)$$

From (8), it is obvious that the wavelength ($\lambda^{[\ell]}$) of the LEO satellite affects the calculation of $\mathbf{G}$ matrix, hence DDOP values, which, in turn, affect the positioning performance of UT. Doppler-shift measurement error also plays a key role in positioning performance. The effect of measurement error (ϵ_{f_d}) in the positioning performance of UT is as follows [3].

$$\|\Delta \mathbf{x}_{\mathrm{UT}}\| \geq \frac{\left\| \mathbf{x}_{\mathrm{sat}}^{[\ell]} - \mathbf{x}_{0,\mathrm{ut}} \right\|}{2 \left\| \mathbf{v}_{\mathrm{sat}}^{[\ell]} \right\|} \times \lambda^{[\ell]} \times \epsilon_{f_d}. \quad (9)$$

Thus, with the same Doppler-shift measurement error, the positioning error of UT is proportional to the satellite-to-UT distance and the wavelength of the transmitted signals, and inversely proportional to the satellite velocity. The higher the transmit frequency, the lower the signal wavelength, resulting in lower positioning error, and vice versa. Considering the satellite altitude of 600 km and with a Doppler shift measurement error of 0.6 Hz, the minimum position error of UT is 4.53 m, 0.60 m, and 0.35 m for the transmit frequency bands of L1 (1.575 GHz), Ku (12 GHz), and K (20 GHz), respectively.

IV. Results and Discussion

This section presents a comparison of Doppler-shift positioning errors for UT using different transmit signal frequencies. The positioning algorithm is described in [1]. The MATLAB communication Toolbox, Starlink TLE files, and Sionna ray tracing are used for simulation evaluation [4]. The evaluation results with different transmit frequencies are tabulated in Table 1.

Table 1. Comparison of positioning results.

Signals transmit frequency	Positioning RMSE (m)			
	North	East	Up	3D
L1 band (1.575 GHz)	1.68	3.45	3.25	5.04
Ku band (12 GHz)	0.33	0.67	0.66	1.00
K band (20 GHz)	**0.24**	**0.50**	**0.51**	**0.76**

From table 1 it is obvious that higher the carrier frequency results in less positioning error, making it more suitable for Doppler shift-based positioning.

IV. Conclusion

LEO satellites are advantageous for their higher signal strength and are resilient to jamming and spoofing over the GNSS. Doppler shift from LEO satellites has significant potential in a GNSS-denied environment, such as indoor and urban areas. This paper presents an analysis of Doppler positioning with different transmit frequency bands. The evaluation results demonstrate that the K band frequency is more suitable for better positioning of the Doppler shift than the L1 band frequency.

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