

# Dual-Mode Chirp-Pair Indexing for AFDM-ISAC: 4-Bit Embedding With Fixed-Reference Sensing

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## Abstract

This work introduces a radar-centric, affine frequency division multiplexing dual-mode index modulation (AFDM-DM) technique that improves index throughput while maintaining sensing ambiguity control. In the proposed 4-bit dual-mode system, each AFDM block chooses one of 16 radar-screened pre- and post-chirp pairs arranged into two modes. Hierarchical index detection (a coarse mode determination followed by an intra-mode search) in the structured codebook reduces the number of hypothesis evaluations compared to exhaustive correlation across all possibilities. A unitary chirp modulation operator generates AFDM, and sensing uses a fixed reference and a typical matched-filter range-Doppler processing. We measure the communication-sensing trade-off using BER and index error rate and assess sensing reliability using detection probability under a controlled false-alarm constraint. Sidelobe metrics (PSLR and ISLR) and representative range-Doppler maps verify radar invariance across all indices. Simulation results show that the dual-mode AFDM-DM has greater index capacity with reasonable receiver complexity and radar-compliant sensing performance.

## 1. Introduction

Integrated sensing and communication (ISAC) combines a data link with a sensing waveform; therefore, any increase in throughput must not compromise ambiguity resolution or target detection reliability [1]. Affine frequency-division multiplexing (AFDM) is inherently well-suited to doubly dispersive channels due to its pre/post-chirp factors (which facilitate controllable delay-Doppler shaping) while preserving a unitary modulation operator [2, 3]. Nevertheless, radar-compliant AFDM-DM is often limited to a small number of index bits per block: expanding the chirp-pair set may increase sidelobes and, in practical applications, necessitate expensive, exhaustive hypothesis testing at the receiver. In multicarrier communications, dual-mode index modulation has demonstrated an ability to enhance index capacity while maintaining structured detection, for example, by organizing candidates into modes that facilitate reduced-search receivers [4].

This work proposes a 4-bit dual-mode AFDM-IM architecture that assigns 16 chirp pairs into two modes of 8 pairs each, enabling a two-stage detector (mode decision followed by index selection) while preserving fixed-reference matched-filter range-Doppler sensing. The main contribution is embedding 4 index bits into AFDM without reducing sidelobe-controlled sensing as the codebook expands, using a radar-compliant dual-mode chirp-pair codebook and hierarchical index detection that boosts index capacity while remaining practical for receivers.

## 2. System Model

We consider a monostatic ISAC node that transmits one AFDM block of length  $N$  per signaling interval. Each block carries (i)  $N$  information symbols  $\mathbf{s} \in \mathcal{S}^N$  drawn from a complex constellation  $\mathcal{S}$  and (ii) a 4-bit index word  $\mathbf{b} = [b_1, b_2, b_3, b_4]^T \in \{0, 1\}^4$  that selects the chirp parameters of the AFDM modulator. To increase index capacity while preserving radar-compliant ambiguity behavior, we employ a dual-mode codebook in which the first bit selects the mode and the remaining bits select an intra-mode entry:

$$m = b_1, \quad q = 1 + 4b_2 + 2b_3 + b_4, \quad (1)$$

where  $m \in \{0, 1\}$  and  $q \in \{1, \dots, 8\}$ . Each mode is associated with a radar-screened chirp-pair set  $\mathcal{C}^{(m)} = \{(c_1^{(m)}[q], c_2^{(m)}[q])\}_{q=1}^8$ , where  $c_1$  and  $c_2$  denote the pre- and post-chirp parameters, respectively. Given  $(m, q)$ , the selected chirp pair is  $(c_1, c_2) = \mathcal{C}^{(m)}[q]$ . The AFDM transmit vector  $\mathbf{x} \in \mathbb{C}^N$  is generated by a unitary modulation operator  $\mathbf{A}(c_1, c_2) \in \mathbb{C}^{N \times N}$ :

$$\mathbf{x} = \mathbf{A}(c_1, c_2)\mathbf{s}, \quad \mathbf{A}^H(c_1, c_2)\mathbf{A}(c_1, c_2) = \mathbf{I}_N, \quad (2)$$

where  $(\cdot)^H$  denotes the Hermitian transpose and  $\mathbf{I}_N$  is the  $N \times N$  identity matrix. The unitary property ensures energy preservation across indices and supports fair comparisons of communication and sensing performance.

The received communication vector is modeled as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \quad (3)$$

where  $\mathbf{H} \in \mathbb{C}^{N \times N}$  denotes the effective doubly dis-

persive channel operator (capturing delay-Doppler coupling within an AFDM block) and  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$  is circularly symmetric complex Gaussian noise with variance  $\sigma^2$  per component.

Index recovery is performed using a hierarchical (two-stage) detector to reduce search complexity. For each hypothesis  $(m, q)$ , define the residual metric:

$$\Lambda_{m,q} = \|\mathbf{y} - \mathbf{H}\mathbf{A}(c_1, c_2)\hat{\mathbf{s}}_{m,q}\|_2^2, \quad (c_1, c_2) = \mathbf{C}^{(m)}[q], \quad (4)$$

where  $\hat{\mathbf{s}}_{m,q}$  is the symbol estimate under hypothesis  $(m, q)$  (e.g., linear equalization followed by constellation slicing) and  $\|\cdot\|_2$  denotes the Euclidean norm. The mode decision is obtained by selecting the mode that yields the smallest within-mode residual,  $\hat{m} = \arg \min_{m \in \{0,1\}} \min_{q \in \{1, \dots, 8\}} \Lambda_{m,q}$ , followed by intra-mode selection  $\hat{q} = \arg \min_{q \in \{1, \dots, 8\}} \Lambda_{\hat{m},q}$ . This separation makes it natural to report both the BER for  $\mathbf{s}$  and the index error probability  $P_e^{\text{index}} = \Pr\{(\hat{m}, \hat{q}) \neq (m, q)\}$ , thereby distinguishing symbol errors from chirp-indexing errors.

For sensing, the node retains a fixed reference block  $\mathbf{x}_{\text{ref}}$  (known to the receiver) and applies standard matched-filter processing. Specifically, over a coherent processing interval (CPI) of  $M$  consecutive AFDM blocks, the receiver performs correlation with  $\mathbf{x}_{\text{ref}}$  in fast-time and an  $M$ -point FFT across slow-time to form a range-Doppler map; the detection statistic  $T(\cdot)$  may be taken as the peak magnitude (or another prescribed functional) of this map. Let  $\gamma$  be a threshold. The sensing operating point is characterized by  $P_D = \Pr\{T > \gamma \mid \mathcal{H}_1\}$  and  $P_{FA} = \Pr\{T > \gamma \mid \mathcal{H}_0\}$ , where  $\mathcal{H}_1$  and  $\mathcal{H}_0$  denote the target-present and target-absent hypotheses, respectively. In addition to  $P_D$  under controlled  $P_{FA}$ , we verify radar invariance across all 16 indices by reporting sidelobe metrics (PSLR/ISLR) and representative range-Doppler maps, and we quantify implementation cost via runtime (or tested hypotheses) relative to exhaustive search.

### 3. Simulation Results

Fig. 1 compares communication reliability for AFDM-IM and the proposed AFDM-DM. Both BER and index error decrease nearly exponentially with SNR, confirming stable operation under improved link quality. AFDM-DM consistently achieves lower error floors: across the entire SNR range, its BER curve lies below that of AFDM-IM, and its index error shows a clear margin, indicating more reliable index recovery and, consequently, fewer data errors. Fig. 2 reports sensing performance in terms of  $P_D$  versus SNR under various false-alarm constraints. As shown, relaxing the threshold (larger  $P_{FA}$ ) yields slightly higher  $P_D$  at low-to-mid SNR, while all curves converge to  $P_D \approx 1$  beyond  $\sim 12$ – $14$  dB.

### 4. Conclusion

This work introduced a dual-mode AFDM framework that improves index and bit reliability while maintaining strong sensing performance under controlled false-alarm constraints. Results verified consistent BER, index-error reduction, and rapid convergence to high detection probability as SNR increases. Future work will

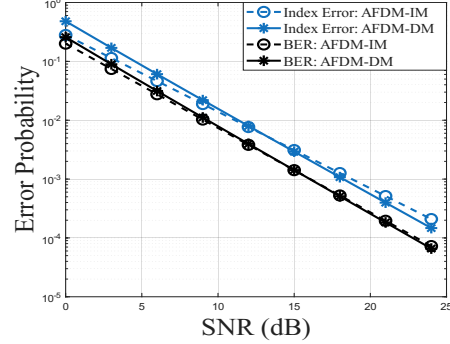


Figure 1. BER performance comparison of AFDM-IM and the proposed AFDM-DM.

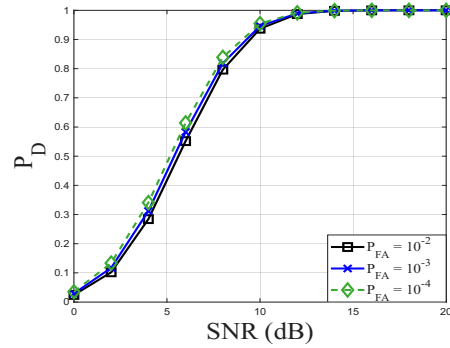


Figure 2. Detection probability  $P_D$  versus SNR for various false-alarm constraints.

extend the design to realistic multipath delay-Doppler channels in multi-target scenarios and develop learning-aided codebook adaptation with complexity-aware detection.

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