Portfolio Optimization with QAOA

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***Abstract*—Portfolio Optimization (PO) is a foundational concept in the trading and investment industry, previously approached through classical methods such as Modern Portfolio Theory (MPT). However, as financial markets grow in complexity and size, these traditional methods face significant computational limitations. To overcome these challenges, machine learning-based approaches have gained popularity. Recently, the advent of quantum computing has sparked interest in quantum-inspired algorithms for portfolio optimization. In this paper, we propose a novel approach utilizing the Quantum Approximate Optimization Algorithm (QAOA), a quantum-based optimization technique, to solve the portfolio optimization problem formulated as a Quadratic Unconstrained Binary Optimization (QUBO) problem.**

***Index Terms*—Quantum computing, portfolio optimization, QAOA, variational algorithms, QUBO.**

I. Introduction

Portfolio Optimization (PO) is a core task in financial decision making, centered on allocating assets to maximize return and minimize risk. Introduced by Nobel laureate Harry Markowitz as Modern Portfolio Theory (MPT) [1], it remains foundational in investment strategy. However, as financial markets expand and data complexity increases, traditional optimization methods struggle with scalability and efficiency [2].

To overcome these limitations, researchers have begun exploring advanced computational techniques, including machine learning and, more recently, quantum computing. In this work, we investigate the Quantum Approximate Optimization Algorithm (QAOA), a promising quantum algorithm for tackling combinatorial problems, by formulating the PO problem as a Quadratic Unconstrained Binary Optimization (QUBO) model. This approach aims to assess QAOA's potential for achieving efficient, constraint-aware portfolio optimization. The rest of the paper is arranged as follows. Section II introduces the related works. Section III presents the problem formulation of our proposed algorithm and its solution. Finally, Section IV provides the conclusion of our work.

II. Related Works

Recent advances in quantum computing have driven research into Quantum Machine Learning (QML), which utilizes quantum systems for enhanced data processing capabilities [3], [4]. In finance, quantum-inspired methods have been explored for tasks such as asset allocation, risk modeling, and pricing.

QAOA has emerged as a promising algorithm for portfolio optimization, particularly due to its ability to solve combinatorial problems efficiently [5]. Other notable approaches include Quantum Support Vector Machines (QSVM) for financial classification problems [6], and quantum annealing for solving QUBO-formulated financial models. Despite these developments, most studies remain in the early stages due to hardware constraints and limited scalability.

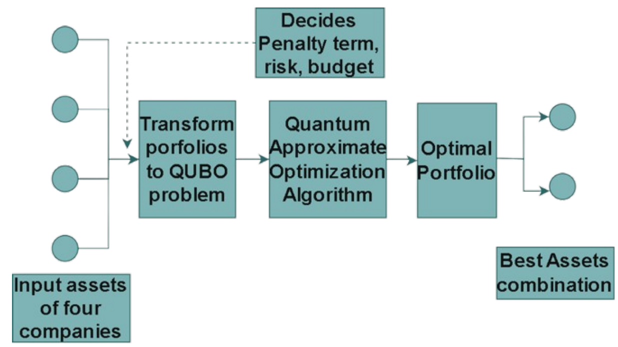

Fig. 1. QAOA algorithm for selecting best optimal portfolio

Building on these foundational efforts, this paper explores the practical readiness of QAOA for realistic, constraint-aware portfolio optimization tasks.

III. Problem Formulation and Results

This section explains the detailed setup of our model and approach to achieve the optimal portfolios from four companies—"Apple", "Microsoft Corporation", "Alphabet", and "Amazon". The historical stock price data is retrieved by using `yfinance` API [7].

A. Problem Setup

We consider a portfolio of n assets (in our case, n = 4). The historical daily closing prices from January 1, 2018, to February 1, 2025, are retrieved using the `yfinance` library. These fetched prices are utilized to compute the daily return matrix R as described in eq. (1).

$$R_{ij} = \frac{P_{ij} - P_{i(j-1)}}{P_{i(j-1)}}, \tag{1}$$

where P_{ij} denotes the closing price of asset i at time j. From the return matrix, the expected return vector and covariance matrix are computed as follows.

$$\boldsymbol{\mu} = \mathbb{E}\{R\}, \quad \boldsymbol{\Sigma} = \mathbb{C}\mathrm{ov}\{R\}. \tag{2}$$

Here, $\boldsymbol{\mu}$ denotes the average return of each asset and $\boldsymbol{\Sigma}$ captures the variance and pairwise correlation of the returns, representing portfolio risk.

B. QUBO Formulation

The portfolio optimization problem is formulated as a Quadratic Unconstrained Binary Optimization (QUBO) problem, suitable for both classical and quantum solution methods. Let $x_i \in \{0,1\}$ be a binary decision variable, where $x_i = 1$ denotes the asset i is selected, and $x_i = 0$ otherwise.

The QUBO objective function is:

$$\min_{\mathbf{x}\in\{0,1\}^n} \Big(-\boldsymbol{\mu}^T\mathbf{x} + q\cdot\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x} + \lambda\cdot\big(\sum_{i=1}^{n} x_i - B\big)^2\Big) \tag{3}$$

where q is a risk aversion coefficient (set to 0.5), B is the maximum number of assets allowed in the portfolio, λ is a penalty factor [8]. The objective function balances expected return and risk while softly enforcing budget constraint by using the penalty term $\lambda\cdot\left(\sum_{i=1}^{n} x_i - B\right)^2$ on the number of selected assets, which eventually restricts choosing the exact number of assets.

C. Solution Approaches

We employ two approaches to solve the QUBO problem:

1) Classical Solver: The `NumPyMinimumEigensolver` from Qiskit's optimization module is utilized to compute the exact ground state of the QUBO Hamiltonian. This serves as a classical benchmark for performance comparison of our QAOA.

2) Quantum Solver: The Quantum Approximate Optimization Algorithm (QAOA) utilized in this work is a hybrid quantum-classical variational algorithm that alternates between applying a cost Hamiltonian and a mixer Hamiltonian. The setup of the QAOA framework for portfolio optimization is illustrated in Fig. 1. The algorithm seeks to minimize the QUBO objective by tuning variational parameters γ and β (see Algorithm 1):

$$F(\gamma,\beta) = \langle\psi(\gamma,\beta)|H_{\text{QUBO}}|\psi(\gamma,\beta)\rangle, \tag{4}$$

where H_{QUBO} is the cost Hamiltonian and F denotes the fidelity, which computes how closely the state $\psi(\gamma,\beta)$ aligns with the true ground state of H_{QUBO}. We use three layers of QAOA (`reps=3`) and the constrained optimization by linear approximation (COBYLA) optimizer to find the best parameters. The quantum circuit is executed on qiskit_aer simulator.

Algorithm 1 Portfolio Optimization via QUBO and QAOA

Input: Ticker symbols, date range, risk factor α, budget B, penalty λ
Output: Optimal portfolio, mean return, and standard deviation
Retrieve historical closing prices using `yfinance`
Compute daily returns R, expected returns μ, and covariance matrix Σ
Formulate QUBO model from the mean-variance objective
Convert QUBO into a quadratic program using `PortfolioOptimization`
Solve using:
- `NumPyMinimumEigensolver` as a classical baseline
- QAOA (`reps`=3) with COBYLA optimizer via `MinimumEigenOptimizer`

for each of 1000 Monte Carlo iterations **do**
    Set seed and reinitialize QAOA
    Run QAOA and obtain output distribution
    Extract bitstring with highest probability
    Evaluate objective function value for selected portfolio
end for
Compute statistics: mean return, standard deviation
return optimal portfolio and associated metrics =0

RESULTS

The classical brute-force method evaluated all possible portfolio combinations and identified the global optimal value of -0.0017 by using classical brute-force method. To evaluate QAOA's efficiency, we compared its output using 1000 shots. The results are summarized in Table I.

TABLE I
COMPARISON OF QAOA PERFORMANCE WITH CLASSICAL METHOD

Method / Shots	Selected Portfolio	Objective Value
Classical (Brute-force)	[1, 1, 0, 0]	-0.0017
QAOA (1000 shots)	[1, 1, 0, 0]	-0.0017

Even with as few as 1000 sampling shots, QAOA consistently identified the correct portfolio configuration. While the objective values differ slightly from the classical optimum due to quantum approximation and sampling variability, they remain remarkably close.

However, brute-force methods scale poorly with problem size: the number of portfolio combinations grows exponentially as 2^n for n assets, quickly rendering classical enumeration computationally infeasible. These results highlight the strength of QAOA in providing high-quality, near-optimal solutions with dramatically fewer evaluations. This efficiency makes QAOA a scalable and promising alternative to brute-force methods, especially as the dimensionality of the problem increases.

IV. CONCLUSION

In this work, the potential application of QAOA for portfolio optimization in financial industry is explored. The simulation tasks are performed by reformulating the optimization task as a QUBO model with QAOA-compatible quantum circuits. Since the experiment is based on a Monte Carlo simulation involving 1,000 randomly sampled weight distributions, the goal is to identify the optimal combination of weights that yields the highest expected return with minimal associated risk. Although current hardware limitations restrict scalability, our findings indicate that QAOA holds promise for solving complex financial optimization tasks as quantum hardware matures. Future work may include improving constraint encoding techniques, applying hybrid quantum-classical optimization frameworks, and extending this approach to dynamic or multi-period portfolio settings.

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