

# Empirical Analysis of Copy-Number Effects in Helstrom Quantum Classification for Titanic Survival

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**Abstract**—We investigate the practical gains of multi-copy quantum discrimination on a real-world dataset. Using the Titanic survival data, we embed every two-feature pair into a qubit space via three schemes: Amplitude, phase, and angle encoding, and form class density operator by averaging. For each encoder, we apply the Helstrom measurement to  $n$ -copy tensor products ( $1 \leq n \leq 8$ ), recording test accuracy and the commutator between two density classes. Contrary to asymptotic theory, our experiments reveal no strong copy-dependent accuracy gains ( $r = 0.042$ ) and only a weak positive link between non-commutativity of the class density operator and accuracy ( $r = 0.174$ ). Surprisingly, the commutator norm itself slightly decreases with  $n$  ( $r = -0.114$ ). These results show that, in low-dimensional, finite-sample settings, simply increasing the number of copies does not guarantee improved classification performance.

## I. INTRODUCTION

Quantum hypothesis testing provides a rigorous framework for binary classification by leveraging the Helstrom measurement [1], which is provably Bayes-optimal for distinguishing between two quantum states. Theoretically, when the two density operators do not commute, preparing  $n$  copies of each state in the asymptotic limit yields an exponential decay of the minimum error probability—known as the quantum Chernoff bound [2]—implying that additional copies should monotonically improve classification accuracy. However, real-world datasets often violate these ideal conditions [3], and empirical guidance on how copy number, operator non-commutativity, and embedding choice interact in practice remains scarce.

In this paper, we systematically explore these interactions on the classical Titanic survival dataset by evaluating every two-feature combination of passenger attributes using the Helstrom Quantum Classifier (HQC) [4] and, for comparison, the classical algorithms Decision Tree, Random Forest (RF), Logistic Regression (LR), Support Vector Machine (SVM), and k-Nearest Neighbors (k-NN) [5]. For the quantum part, we embed each two-dimensional feature vector into a quantum state via three distinct schemes—amplitude, phase, and angle encoding [3]—and construct class prototypes by simple averaging. For each encoder, we then apply the Helstrom measurement on  $n$ -copy tensor products (with  $n$  ranging from 1 to 8) and record both the test accuracy and the commutator. Finally, we compute the correlations to quantify (a) how non-commutativity grows with  $n$ ,

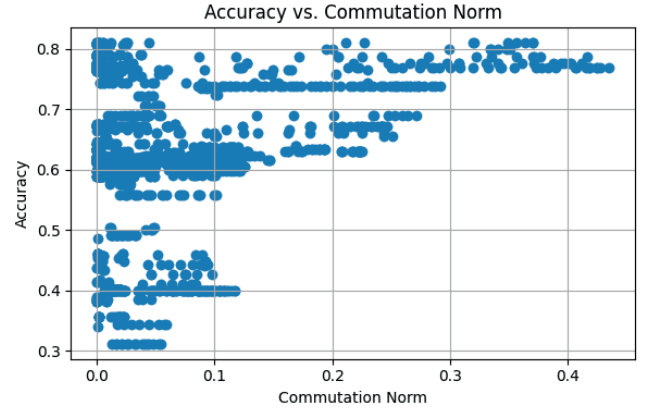


Figure 1: Plot of accuracy vs commutation norm of class density operators of HQC.

(b) how accuracy grows with  $n$ , (c) how accuracy relates to non-commutativity, and (d) how HQC compares to classical algorithm.

Our key empirical findings are that, across hundreds of two-feature tests and all three embeddings, (a) the commutator norm shows a slight, negative correlation with copy number ( $r = -0.114$ , see Fig. 1), (b) accuracy shows virtually no correlation with copy number ( $r = 0.042$ ), (c) accuracy has only a weak positive correlation with non-commutativity ( $r = 0.174$ ), and (d) the HQC achieves accuracy comparable to that of classical algorithms (see Fig. 2). These results challenge the straightforward application of the quantum Chernoff intuition.

## II. QUANTUM HELSTROM BINARY CLASSIFIER

### A. Quantum Data Embeddings

We map each 2D feature  $x = (x_0, x_1)$  to a quantum state by first normalizing [3]

$$u = \frac{x}{\|x\|}, \quad \|x\| = \sqrt{x_0^2 + x_1^2}, \quad (1)$$

and substituting  $\frac{1}{2}I$  if  $\|x\|$  falls below a threshold.

1) *Amplitude Encoding*: Use the real components of  $u$  as amplitudes [3]:

$$|\psi_{\text{amp}}\rangle = u_0 |0\rangle + u_1 |1\rangle, \quad \rho_{\text{amp}} = |\psi_{\text{amp}}\rangle\langle\psi_{\text{amp}}|. \quad (2)$$

2) *Phase Encoding*: Keep  $|u_0|$  and  $|u_1|$  but add a phase  $\phi = \pi u_1$  [3]:

$$|\psi_{\text{ph}}\rangle = u_0 |0\rangle + e^{i\phi} u_1 |1\rangle, \quad \rho_{\text{ph}} = |\psi_{\text{ph}}\rangle\langle\psi_{\text{ph}}|. \quad (3)$$

3) *Angle Encoding*: Interpret  $u$  as spherical angles [4]:

$$\theta = 2 \arccos(u_0), \quad \varphi = \arg(u_0 + i u_1), \quad (4)$$

$$|\psi_{\text{ang}}\rangle = \cos\frac{\theta}{2} |0\rangle + e^{i\varphi} \sin\frac{\theta}{2} |1\rangle, \quad \rho_{\text{ang}} = |\psi_{\text{ang}}\rangle\langle\psi_{\text{ang}}|. \quad (5)$$

### B. Class Density Operators by Simple Averaging

Given a training set  $\{(x_i, y_i)\}$  with binary labels  $y_i \in \{0, 1\}$ , one computes for each class the empirical density matrix [4]

$$\rho_0 = \frac{1}{N_0} \sum_{i: y_i=0} \rho(x_i), \quad \rho_1 = \frac{1}{N_1} \sum_{i: y_i=1} \rho(x_i), \quad (6)$$

where  $N_k$  is the number of samples in class  $k$ . This “prototype” construction requires no iterative parameter fitting—just a weighted average of the embedded states.

### C. Minimum-Error Measurement (Helstrom POVM)

The goal is to decide between the two class states  $\rho_0$  and  $\rho_1$  with lowest possible error, assuming equal prior probabilities. One defines the Helstrom operator  $\Delta$  [1], and diagonalizes it in its eigenbasis. The optimal two-element POVM for binary discrimination is

$$\Delta = \frac{1}{2}(\rho_0 - \rho_1), \quad E_0 = \sum_{\lambda_i > 0} |v_i\rangle\langle v_i|, \quad E_1 = I - E_0, \quad (7)$$

where  $\{\lambda_i, |v_i\rangle\}$  are the eigenpairs of  $\Delta$ . Measuring an unknown state  $\rho(x)$  with this POVM yields outcome 0 (class 0) with probability  $\text{Tr}[\rho(x) E_0]$ , and 1 otherwise. This prescription is known to minimize the average error probability  $P_{\text{err}} = \frac{1}{2}(1 - \|\Delta\|_1)$ , where  $\|\Delta\|_1 = \sum_i |\lambda_i|$  is the trace norm of  $\Delta$ .

### D. Multi-Copy Extension

To exploit an exponential convergence of error, one can prepare  $n$  identical copies of each embedded state and form [2]

$$\rho_0^{\otimes n}, \quad \rho_1^{\otimes n}, \quad \Delta^{(n)} = \frac{1}{2}(\rho_0^{\otimes n} - \rho_1^{\otimes n}). \quad (8)$$

Repeating the POVM construction on the  $2^n$ -dimensional space yields an error probability scaling as  $P_{\text{err}}^{(n)} \approx \exp(-\xi n)$ , where  $\xi > 0$  is the quantum Chernoff exponent. The classification proceeds by computing  $\text{Tr}[\rho(x)^{\otimes n} E_0^{(n)}] \geq 0.5$ .

### E. Non-Commutativity and Encoding Quality

Because  $\rho_0$  and  $\rho_1$  need not commute, the Helstrom POVM can exploit genuinely quantum interference effects. In practice, one can quantify the degree of non-commutativity by the Frobenius norm [2] of their commutator

$$\|[\rho_0, \rho_1]\|_F = \|\rho_0 \rho_1 - \rho_1 \rho_0\|_F. \quad (9)$$

A small commutator norm indicates nearly classical (commuting) prototypes—often leading to limited quantum advantage—whereas larger norms signal richer quantum discrimination potential.

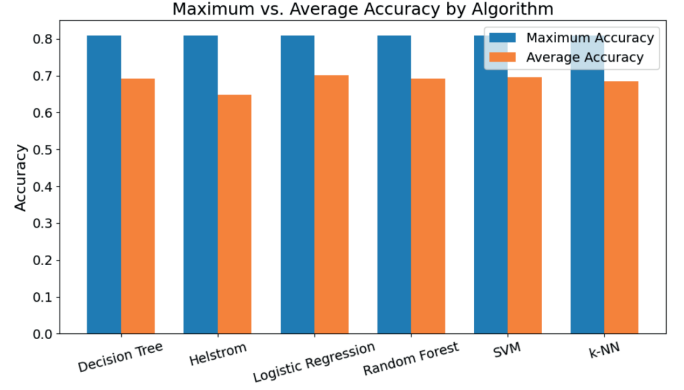


Figure 2: Maximum (blue) and average (orange) accuracies for each algorithm.

## III. CONCLUSION

Our extensive empirical study reveals that the theoretical promise of multi-copy Helstrom discrimination—namely, monotonic accuracy gains with increasing  $n$ —does not automatically materialize on real data. In particular, amplitude encoding yields no clear benefit beyond the single-copy case, while phase and angle embeddings produce modest improvements up to a small optimal  $n$  before plateauing. The near-zero correlation between accuracy and copy number ( $r = 0.042$ ) and only a weak link to non-commutativity ( $r = 0.174$ ) indicate that additional copies can amplify sampling noise without unlocking proportional quantum advantage unless the embedding induces sufficiently strong non-commuting class states. As future work, we plan to further investigate regions where the class density operators are nearly commuting, and to implement Helstrom multi-copy classification on an FPGA to accelerate the diagonalization process, as in [6].

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