Bayesian One-Shot Quantum State Discrimination under Measurement Constraints

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***Abstract*—Quantum state discrimination is a foundational task in quantum information processing. In conventional scenarios, multiple identical copies of a quantum state are available, allowing high-confidence decision-making via repeated measurements. However, practical constraints in quantum sensors, low-latency communication, and embedded quantum devices may allow only a single measurement per state. In this paper, we investigate the one-shot quantum state discrimination problem: given a single projective measurement on an unknown qubit state chosen from a known set, how accurately can the state be identified? We focus on binary discrimination between two pure states, examining the influence of state overlap and prior distribution on classification accuracy. Using Bayesian optimal decision rules, we show that meaningful accuracy can be achieved even in the one-shot regime, especially with favorable priors or state separation. This work highlights the viability of low-resource quantum inference strategies and their relevance to practical quantum technologies.**

***Index Terms*—Quantum state discrimination, one-shot measurement, Bayesian inference, quantum decision theory, low-resource quantum systems.**

I. Introduction

Quantum state discrimination is a fundamental problem in quantum information theory with implications for quantum communication [1], [2], quantum sensing [3], and quantum cryptography. It is the task of identifying which quantum state from a known set was prepared, using measurement data. The optimal performance in such tasks is constrained by the indistinguishability of non-orthogonal quantum states, which is a key non-classical feature of quantum mechanics [4], [5], [6].

In most theoretical treatments and practical implementations, discrimination strategies assume access to multiple copies of the quantum state. Repeated measurements allow inference through statistical estimation and error mitigation. However, in emerging quantum technologies such as energy-efficient quantum sensors, mobile quantum platforms, and fast quantum communication protocols, resources are often limited [7]. Coherence times may be short, power consumption must be minimized, and only one-shot measurements may be feasible.

This paper investigates the problem of *one-shot quantum state discrimination*. We assume that a quantum system is prepared in one of two known pure states, drawn from a known prior distribution. A single projective measurement is allowed, and the observer must classify the state based on this outcome alone. The challenge is to determine the best possible decision rule under these conditions and to quantify how performance varies with the geometry of the state space and the prior information.

Our work focuses on binary pure-state discrimination, employing projective measurements in the computational basis. We use Bayesian decision theory to derive an optimal one-shot classifier and perform a detailed numerical study of its performance across a grid of state overlaps and prior probabilities. We produce a two-dimensional heatmap showing the variation of classification accuracy and analyze the limiting behavior in symmetric and asymmetric regimes.

II. Problem Setup

Let $\mathcal{H} = \mathbb{C}^2$ denote the Hilbert space of a single qubit. We define two pure states parametrized by an angle θ:

$$|\psi_1\rangle = \cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle\,, \tag{1}$$
$$|\psi_2\rangle = \sin(\theta)\,|0\rangle + \cos(\theta)\,|1\rangle\,. \tag{2}$$

Here, $\theta \in (0, \pi/2)$ determines the degree of non-orthogonality. The inner product between the states is:

$$\langle\psi_1|\psi_2\rangle = \sin(2\theta), \tag{3}$$

and hence the fidelity between them is $F = |\langle\psi_1|\psi_2\rangle|^2 = \sin^2(2\theta)$.

The task is to identify the state after a single measurement. We assume a projective measurement in the standard basis $\{|0\rangle, |1\rangle\}$, which yields outcome $m \in 0, 1$. The measurement statistics are:

$$P(0|\psi_1) = \cos^2(\theta), \qquad P(0|\psi_2) = \sin^2(\theta), \tag{4}$$
$$P(1|\psi_1) = \sin^2(\theta), \qquad P(1|\psi_2) = \cos^2(\theta). \tag{5}$$

The prior probabilities are $P(\psi_1) = p$ and $P(\psi_2) = 1 - p$. Using Bayes' theorem [8], the posterior is:

$$P(\psi_i|m) = \frac{P(m|\psi_i)P(\psi_i)}{P(m)}, \tag{6}$$

where the marginal $P(m)$ ensures normalization. The Bayes-optimal decision is to select the state with the higher posterior probability.

III. Bayesian One-Shot Classifier

The decision rule is:

$$\text{Predict } \psi_1 \text{ if } P(\psi_1|m) > P(\psi_2|m), \tag{7}$$

and vice versa. This rule minimizes the expected classification error. Since we use fixed measurement operators, the optimality is only within the class of classifiers using a single computational basis measurement.

We note that for non-orthogonal states, a single projective measurement cannot achieve perfect discrimination. The Helstrom bound [9] provides a lower bound on error probability:

$$P_e^* = \frac{1}{2}\left(1 - \sqrt{1 - 4p(1-p)|\langle\psi_1|\psi_2\rangle|^2}\right), \tag{8}$$

but this assumes optimal measurements, whereas we restrict ourselves to a single fixed basis.

IV. Simulation Methodology

To systematically evaluate the performance of the Bayesian one-shot classifier, we implemented a Monte Carlo simulation across a dense grid of state parameters. The simulation iterates over a 500×500 grid of (θ, p) pairs, where θ controls the overlap between the two quantum states and p denotes the prior probability of state $|\psi_1\rangle$. For each configuration, 1000 independent trials are conducted. In each trial, a quantum state is sampled according to the prior distribution, a projective measurement is simulated in the computational basis, and the Bayes-optimal decision rule is applied to classify the state. The proportion of correctly classified outcomes provides an empirical estimate of classification accuracy. The complete procedure is summarized in Algorithm 1, which captures the logic used to generate the performance heatmap presented in the results section.

Algorithm 1 Bayesian One-Shot State Discrimination Simulation

for all pairs (θ, p) in 500×500 grid **do**
  Set accuracy counter $A \leftarrow 0$
  for $i = 1$ to 1000 **do**
    Sample true state $\psi \in \{\psi_1, \psi_2\}$ using prior p
    Perform projective measurement in the computational basis
    Compute likelihoods $P(m|\psi_i)$ and posteriors $P(\psi_i|m)$
    Predict state $\hat{\psi}$ using Bayes-optimal rule
    if $\hat{\psi} = \psi$ **then**
      Increment accuracy counter $A \leftarrow A + 1$
    end if
  end for
  Compute accuracy $a = A/1000$
  Record (θ, p, a)
end for

V. Results

A. Heatmap Visualization

The heatmap in Figure 1 shows classification accuracy as a function of the state angle θ and the prior probability p. Accuracy is lowest when $\theta \approx 45°$ and $p = 0.5$, where the states are maximally overlapping and the prior is uninformative. Accuracy improves as the states become more distinguishable (i.e., as θ approaches 0° or 90°) or as the prior becomes biased toward one state.

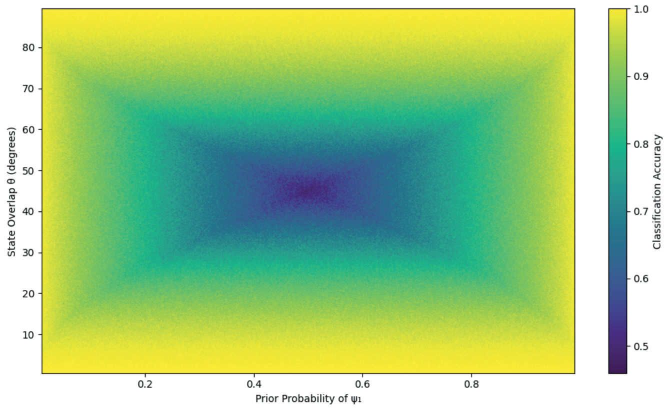

Fig. 1. Classification accuracy as a function of prior $P(\psi_1)$ and state overlap angle θ (degrees).

The worst-case performance occurs near $\theta \approx 45°$ (maximal overlap) and $p = 0.5$ (uniform prior), where the states are nearly indistinguishable and the classifier reduces to random guessing. As θ increases toward 0°, the states become orthogonal, enabling nearly perfect classification accuracy.

B. Selected Configuration Comparison

To complement the heatmap visualization, Table I reports classification accuracy for a few representative (θ, p) settings. These examples illustrate specific points within the performance landscape and provide numerical confirmation of the general trends. Notably, configurations with either highly biased priors or state angles far from maximal overlap (i.e., away from $\theta = 45°$) yield significantly higher accuracy. Conversely, when both the states are nearly indistinguishable and the prior is uninformative, performance approaches the level of random guessing.

TABLE I
One-Shot Accuracy for Selected Configurations

θ (°)	$P(\psi_1)$	$P(\psi_2)$	Accuracy
22.5	0.5	0.5	0.860
22.5	0.9	0.1	0.894
45.0	0.5	0.5	0.495
45.0	0.9	0.1	0.903

VI. Discussion

The results reveal the dual role of geometric distinguishability and prior asymmetry. The classifier leverages both features: geometric separation allows more informative measurement outcomes, while skewed priors reduce uncertainty about the state.

Interestingly, even in the one-shot regime with a fixed measurement basis, the Bayes-optimal rule significantly outperforms random guessing for many configurations. This suggests that in low-latency or resource-constrained environments, carefully designed prior distributions and state preparations can enable effective quantum inference with minimal measurement.

VII. Conclusion and Future Work

We explored the limits of quantum state discrimination under the extreme constraint of one-shot measurement. Our simulation-based analysis shows how prior probabilities and state geometry interact to shape classification accuracy. A Bayes-optimal decision rule can achieve surprisingly good performance, particularly with asymmetric priors or near-orthogonal states.

This study motivates several directions for future work, including:

- Extending to mixed states and realistic noise models.
- Investigating measurement basis optimization under single-shot constraints.
- Applying adaptive or learned decision rules for multi-class settings.
- Incorporating quantum circuits or NISQ devices [10] in real-time discrimination tasks.

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