Optimization and Analysis of the embedding of surface code on the heavy-hexagon lattice

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Heavy-Hexagon 구조에서의 서피스 코드 embedding 최적화 및 분석

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Abstract

This study focuses on the heavy-hexagon lattice, which is the architecture of current IBM quantum computers. As connectivity becomes a challenge in quantum error correction codes, with surface code being a representative one, we optimize an embedding circuit of surface code both using SWAP gates and flag qubits. Based on this, we compare the indexes of the proposed method with the SWAP- and flag-based embeddings in previous works.

Ⅰ. Introduction

Quantum computers hold the promise of solving problems that are beyond the capabilities of classical computers. However, it is sensitive to the errors in the environment when doing quantum computations, therefore quantum error correction (QEC) codes, for example, surface code, are introduced to protect against errors [1].

Previous studies have focused on the surface code on the 2D grid square lattice, which needs next-nearest neighbors to be 4 [2]. To implement on the IBM quantum devices, which have the largest number of qubits, heavy-hexagonal lattice with connectivity being {2,3} has been researched nowadays to reduce crosstalk and gate errors [3]. However, it faces the additional challenge of low connectivity. To tackle this, non-local connection should be considered to embed the surface code. Two types of embeddings, SWAP-based [4] and flag-based [5], are employed to adapt to the connectivity constraints. [4] also proposed an optimized SWAP-based embedding, which uses flag qubits to do the transformation.

In this paper, we introduce a new embedding of surface code on the heavy-hexagon lattice and show the comparison between it and other embeddings proposed before.

Ⅱ. Previous works and proposed embedding

1. Previous works

Based on the IBM quantum computers, physical qubits are located at vertices and edges of a hexagonal lattice, which no longer has 4 next-nearest neighbor qubits like the square structure. As shown in Fig.1 (a), the heavy-hexagon lattice is constructed such that each edge has connectivity $z=2$ and each vertex have connectivity $z=3$.

Thus, due to the limited connectivity, indirect methods should be considered. Here we set distance d=3. Two kinds of embeddings are used in [4]: SWAP-based and flag-based as illustrated in Fig.1 (b).

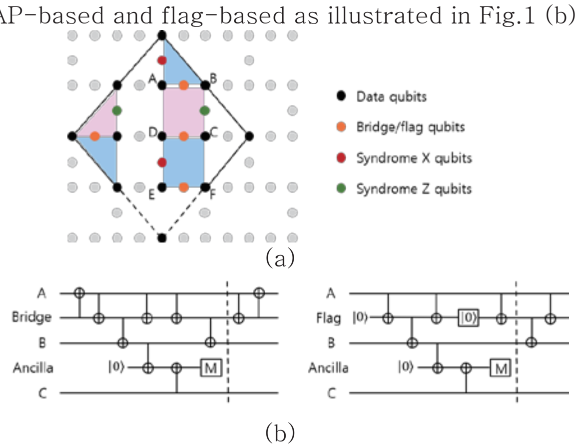


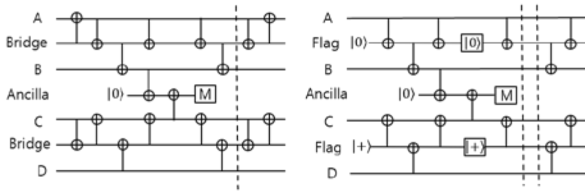
Fig.1 The layout of surface code on the heavy-hexagon lattice (a) and Swap-based (left b) and flag-based (right b) circuits between qubit A, B

As shown in [4], SWAP gates use multiple CNOT operations to exchange the states of two qubits with no information encoded in the bridge qubits. In contrast, flag qubits can simplify the circuit structure, but they require extra resources.

2. Proposed embedding

To better utilize the advantages of the two methods mentioned above, we introduce a new method to make two qubits communicate directly. We apply SWAP-based embedding to the first sub round, where gate errors are dominant, and use flag qubits in the second sub round, which are more sensitive to measurement errors.

We only show the plaquette stabilizer between the qubits A, B, C and D, and the vertex stabilizer would be in the same way, so we omit it here.



(a) (b)

Fig. 2 The first sub round using swap gates (a) and the second sub round using flag qubits (b)

As shown in Fig.2, due to (b) is a time step earlier than (a). here we introduce an idle operation to maintain timing alignment.

Besides the above method, another effective next-nearest-neighbor CX gate can be implemented using 4 CNOT gates, regardless of the state of the middle qubit [6]. We embedded it as shown in Fig. 3.

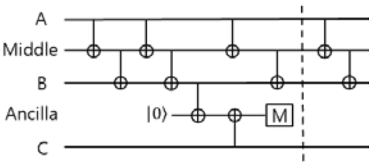


Fig.3 Embedding circuit of surface code based on the lattice in Fig. 1(a)

3. Comparison and analysis

In this part, we compare our proposed embeddings with the others in the previous works, which are shown in table 1 below.

Methods	SWAP-based	Flag-based	Proposed	Method in [6]
Time steps	7	6	7	7
Flag qubits	0	4	2	0
Values of CNOTs	28	20	24	28

Table 1 Comparison between four different embeddings

From the table, we only calculate the CNOTs of the plaquette stabilizers in a single round. It can be observed that our proposed method requires 7 time steps, the same as the SWAP-based embedding. While two additional flag qubits are introduced, the total number remains lower than that of the flag-based embedding. Moreover, our method uses 24 CNOT gates, which is fewer than the number required by the SWAP-based method, though slightly more than the 20 required by the flag-based method.

Ⅲ. Conclusion

In this paper, we introduced a combined embedding of the surface code on the heavy-hexagon lattice and compared this method with original embedding proposed in the former works.

For SWAP-based embedding, it needs more CNOT gates than the other methods. Moreover, the method of using flag qubits requires more depth and measurement. Hence, our proposed method achieves a reduction in the value of CNOT gates by slightly increasing the number of flag qubits.

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REFERENCES

[1] Devitt, Simon J, et al. "Quantum Error Correction for Beginners." Reports on Progress in Physics, vol. 76, no. 7, 20 June 2013, p. 076001.

[2] Fowler, Austin G., et al. "Surface Codes: Towards Practical Large-Scale Quantum Computation." Physical Review A, vol. 86, no. 3, 18 Sept. 2012, p. 032324.

[3] Chamberland, Christopher, et al. "Topological and Subsystem Codes on Low-Degree Graphs with Flag Qubits." Phys. Rev. X, vol. 10, no. 1, 31 Jan. 2020.

[4] Benito, César, et al. "Comparative Study of Quantum Error Correction Strategies for the Heavy-Hexagonal Lattice." Quantum, vol. 9, 6 Feb. 2025, pp. 1623- 1623.

[5] Kim, Younghun, et al. "Design of Quantum Error Correcting Code for Biased Error on Heavy-Hexagon Structure." Quantum Information Processing, vol. 22, no. 6, 3 June 2023.

[6] Hetényi, Bence, and James R. Wootton. "Creating Entangled Logical Qubits in the Heavy-Hex Lattice with Topological Codes." PRX Quantum, vol. 5, no. 4, 3 Dec. 2024.