Dual-User STAR-RIS-Assisted Uplink NOMA: Fairness Optimization

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Abstract

This paper investigates the use of simultaneously transmitting and reflecting intelligent surface (STAR-RIS) to enhance uplink non-orthogonal multiple access (NOMA) for two users. We propose a joint optimization of power allocation and STAR-RIS beamfomrming maximizing the rate fairness among users without resorting to alternating optimization.

Ⅰ. Introduction

The concept of simultaneously transmitting and reflecting intelligent surface (STAR-RIS) have emerged as a key technology for providing 360° coverage through simultaneous transmission and reflection [1]. When combined with non-orthogonal multiple access (NOMA), which improves spectral efficiency and connectivity, this combination shows strong potential for future efficient wireless systems [2]. Previous studies have investigated minimum-rate maximization in RIS-assisted NOMA [3], and our recent work in [4] presented a general multi-user STAR-RIS framework. In this paper we focus on the dual-user case and propose simplified second-order cone program (SOCP) formulation that achieves max-min rate fairness with reduced computational complexity compared to the general multi-user case in [4].

Ⅱ. System Model and Problem Formulation

We consider an uplink NOMA system enhanced by a STAR-RIS, where a single-antenna base station (BS) communicates with two single-antenna users, through STAR-RIS without direct link between users and BS. The STAR-RIS operates in energy splitting mode, with N elements handling both transmission and reflection. For user $q \in \{t, r\}$ the transmission/reflection coefficients are given as:

$$\boldsymbol{\theta}_q = (\sqrt{\beta_{q,1}} e^{j\phi_{q1}},\ \beta_{q,2} e^{j\phi_{q2}}, \cdots, \sqrt{\beta_{q,n}} e^{j\phi_{qn}})^T \tag{1}$$

with t and r denoting transmission and reflection spaces, respectively. The channels between STAR-RIS and BS is denoted by $\boldsymbol{g} \in \mathbb{C}^{N\times 1}$ and that between STAR-RIS and each user q is denoted by $\boldsymbol{f}_q \in \mathbb{C}^{N\times 1}$. For the uplink NOMA, the received signal at the BS is expressed as:

$$y = \sqrt{p_t}\, \boldsymbol{h}_t^T \boldsymbol{\theta}_t s_t + \sqrt{p_r}\, \boldsymbol{h}_r^T \boldsymbol{\theta}_r s_r + n \tag{2}$$

where $\boldsymbol{h}_q = \boldsymbol{g} \circ \boldsymbol{f}_q$, p_q, s_q are the cascaded channel, transmit power and symbol of user q, respectively, with $E[|s_q|^2] = 1$ and $n \sim \mathbb{CN}(0, \sigma^2)$ representing the additive white Gaussian noise at the BS.

We aim to maximize the minimum rate of the two users by jointly optimizing the transmission/reflection coefficients and power allocation as:

$$\max_{\boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{\pi}} \ \min\left(\log_2\left(1 + \frac{p_t \left|\boldsymbol{h}_t^T \boldsymbol{\theta}_t\right|^2}{\pi_r p_r \left|\boldsymbol{h}_t^T \boldsymbol{\theta}_r\right|^2 + \sigma^2}\right), \log_2\left(1 + \frac{p_r \left|\boldsymbol{h}_2^T \boldsymbol{\theta}_r\right|^2}{\pi_t p_t \left|\boldsymbol{h}_1^T \boldsymbol{\theta}_t\right|^2 + \sigma^2}\right) \right) \tag{3a}$$

$$\text{s.t.} \quad |\theta_{\text{tn}}|^2 + |\theta_{\text{rn}}|^2 \le 1, \ \text{n} \in \text{N}, \tag{3b}$$

$$0 \le p_q \le P_q^{\max}, \ q \in \{t, r\} \tag{3c}$$

$$\pi \in \Pi_q \tag{3d}$$

III. Optimization Approach

For a given successive interference cancellation (SIC) order and optimal phase shift $\phi_{q,n} = -\angle h_{q,n}$ along with transmission/reflection amplitude $\boldsymbol{B} = [\boldsymbol{\beta}_t, \boldsymbol{\beta}_r]$, and $\zeta_{qn} = \frac{|h_{qn}|}{\sigma}$ with $\boldsymbol{\zeta_q} = [\zeta_{q1}, \zeta_{q2}, \cdots, \zeta_{qN}]^T$ and by utilizing the monotonic increasing property of the log function, the problem can be reformulated as:

$$\max_{\boldsymbol{B}, \boldsymbol{p}} \ \min\left(\frac{\sqrt{p_t}\, \zeta_t^T \boldsymbol{\beta_t}}{\sqrt{p_r (\zeta_r^T \boldsymbol{\beta_r})^2 + 1}}, \sqrt{p_r}\, \zeta_r^T \boldsymbol{\beta_r} \right) \tag{4a}$$

$$\text{s.t.} \quad \beta_{\text{tn}}^2 + \beta_{\text{rn}}^2 \le 1, \ \beta_{\text{tn}} \ge 0, \ \beta_{\text{rn}} \ge 0, \ \text{n} \in \text{N} \tag{4b}$$

$$0 \le p_q \le P_q^{\max}, \ q \in \{t, r\} \tag{4c}$$

By defining $\boldsymbol{z}_q = \sqrt{p_q}\, \boldsymbol{\beta}_q$ with $\boldsymbol{Z} = [\boldsymbol{z}_t \boldsymbol{z}_r]$ and $\widetilde{\chi}_1 \le \min(f_1, f_2)$, thus the problem can be cast as SOCP and given as:

$$\max_{\boldsymbol{B}, \boldsymbol{Z}, \widetilde{\chi}_1} \ \widetilde{\chi}_1 \tag{5}$$

$$\text{s.t.} \quad \widetilde{\chi}_1 \, \|[\boldsymbol{\zeta}_\mathbf{r}^\mathrm{T} \mathbf{z}_\mathbf{r}, 1]\|_2 \le \boldsymbol{\zeta}_\mathbf{t}^\mathrm{T} \mathbf{z}_\mathbf{t}, \ \widetilde{\chi}_1 \le \boldsymbol{\zeta}_\mathbf{r}^\mathrm{T} \mathbf{z}_\mathbf{r},$$

$$\boldsymbol{z_q} \le \sqrt{P_q^{\max}}\, \boldsymbol{\beta}_q, \ q \in \{t, r\}$$

$$\|[\beta_{tn}, \beta_{rn}]\|_2 \le 1, \quad n \in \boldsymbol{N}$$

The optimal value $\widetilde{\chi}_1^*$ is then obtained by solving a sequence of feasibility problems.

III. Results and Discussion

For our simulations, we adopt a setup similar to [4], including the BS and STAR-RIS deployment, user distribution, and propagation model.

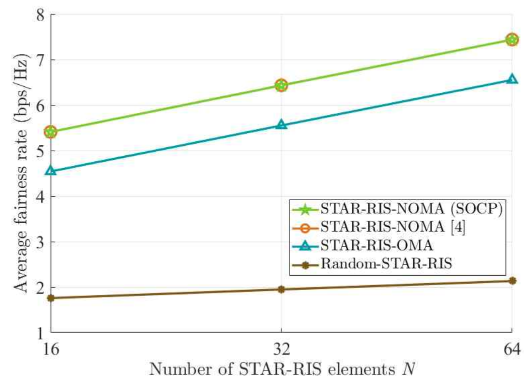

Fig.1. Achievable fairness rate vs number of STAR-RIS elements

We compare our low-complexity SOCP-based STAR-RIS NOMA with the reference STAR-RIS NOMA [4], orthogonal multiple access (OMA), and a random phase‐amplitude STAR-RIS baseline. Our method matches [4] at lower complexity, and both NOMA schemes consistently exceed OMA and the random baseline for all N.

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