

# Outage Performance Analysis of Dual-Hop ISL-Assisted LEO Satellite Communication Systems

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## Abstract

Free-space optical (FSO) enabled inter-satellite links (ISLs), also referred to as inter-satellite optical communication (ISOC) links, are anticipated to be the preferred solution for high-speed and long-distance communication between satellites in advanced sixth-generation (6G) networks. This study investigates the outage performance of dual-hop ISL-assisted low-earth-orbit (LEO) satellite communication systems. The inter-satellite communication is performed over an optical link and is subject to various space channel impairments, including vibrations and perturbations-induced pointing errors, as well as plasma absorption. However, their combined effect is modelled using the Hoyt distribution. Besides, the satellite-ground connection is considered as a radio frequency (RF) link following the Nakagami- $m$  distribution. Based on this, we compute the cumulative distribution function (CDF) and probability density function (PDF) of the end-to-end signal-to-noise ratio (SNR). Thereafter, utilizing PDF and CDF, we derive a series-based expression for probability (OP) employing both heterodyne detection (HD) and intensity modulation with direct detection (IM/DD) techniques under a decode and forward (DF) relaying scheme of the considered system. Additionally, to gain further insights, we emphasize the effects of all considered channel parameters and impairments through numerical results.

## I. Introduction.

In sixth-generation (6G) communication networks, space-air-ground integrated networks (SAGINs) will be essential for providing extensive coverage through both terrestrial and non-terrestrial infrastructures [1]. Free-space optical (FSO)-based inter-satellite links (ISLs), also known as inter-satellite optical communication (ISOC) links, are contemplated as promising technology for SAGINs to offer high-speed, long-distance communications with extensive coverage, enabling both deep-space missions and global coverage. Additionally, ISOC links have several advantages compared to radio frequency (RF)-based ISLs. These benefits include higher data rates, smaller antenna sizes, lighter and more compact terminals, narrower beams, minimal interference, enhanced security, greater directivity, lower transmit power, and operation in unlicensed spectrum. In recent years, extensive research has been carried out by academic and industrial researchers on ISOC [1]. Motivated by the existed literature, we evaluate the outage performance of dual-hop ISL-assisted low earth orbit (LEO) satellite communication systems.

## II. System and Channel Models

In this work, we consider a downlink communication scenario involving two co-orbital satellites in LEO that communicate with each other via an optical link and with the ground using a RF link as shown in Fig. 1. The communication from the satellite (S) to the user (U) occurs in two hops. In the first hop, S transmits an optical signal to the satellite (R) (S-R) over an optical link. In the second hop, an optical signal received at R is converted into an electrical signal, which is then transmitted to U via the RF link (R-U) employing decode-and-forward (DF) relay. Moreover, we assumed that an optical signal is detected at R using either the heterodyne detection (HD) and intensity modulation with direct detection (IM/DD) technique.

### A. S-R Link

When an optical signal propagates through the space, it inevitably experiences fading, distortion, attenuation, and

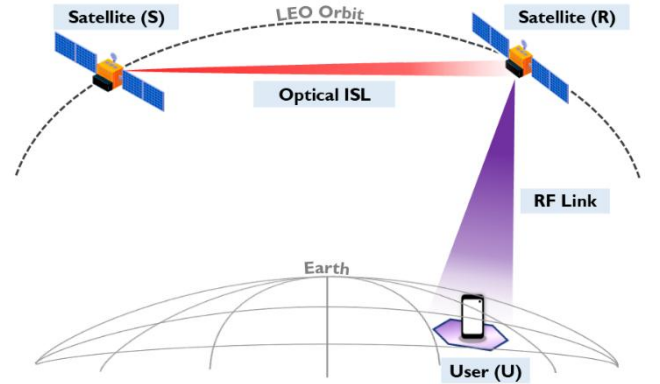


Fig. 1. System model of the dual-hop ISL-assisted LEO satellite communication system.

divergence due to unpredictable space environment conditions. Therefore, optical signals cannot propagate to receiver apertures accurately. Thus, we consider the impact of vibrations and perturbation-induced pointing errors and plasma absorption on the S-R link. Therefore, the combined fading coefficient can be written as

$$h_{SR} = h_p \cdot h_a, \quad (1)$$

where  $h_p$  and  $h_a = \exp(-Ld)$  represent the fading channel coefficient corresponding to vibrations and perturbation-induced pointing errors and plasma absorption, respectively. The optical link length and attenuation coefficient denoted by  $d$  and  $L$ , respectively. The received signal at R can be written as

$$r_{SR} = \sqrt{P_{T_1}} h_{SR} s + n_{SR}. \quad (2)$$

In Eq. (2),  $\sqrt{P_{T_1}}$  denotes transmitted power,  $s$  is modulated signal, and  $n_{SR}$  shows additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_{SR}^2$ . The combined probability density function (PDF) of  $h_{SR}$  is derived as follows:

$$f_{h_{SR}}(h) = \frac{w_{zeq}^2 \sqrt{\eta}}{4\sigma_2^2 h} \left( \frac{h}{h_a A_0} \right)^{\frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2}} I_0 \left[ \frac{(1-\eta)w_{zeq}^2}{8\sigma_2^2} \ln \left( \frac{h_a A_0}{h} \right) \right]. \quad (3)$$

In Eq. (3),  $0 \leq h \leq h_a A_0$ ,  $w_{zeq}$  denotes equivalent beam waist, and  $A_0$  indicates the amount of the collected power. Further,  $\eta = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$  where  $\sigma_1^2$  and  $\sigma_2^2$  are variances of radial displacement resulting from perturbations and vibrations. Moreover,  $I_0(\cdot)$  symbolizes modified Bessel function of the first kind of zero order.

### B. R-U Link

The received signal at U when the DF relaying scheme is utilized at R can be expressed as

$$r_{RU} = \sqrt{P_{T_2} P_{L_0}} h_{RU} s' + n_{RU}, \quad (4)$$

where  $P_{T_2}$  is transmitted power by R and  $P_{L_0}$  denotes pathloss between R and U. Further,  $h_{RU}$  characterizes the RF link channel fading coefficient. Moreover,  $n_{RU}$  refers as the AWGN with zero mean and variance  $\sigma_{n_{RU}}^2$ .

## III. End-to-End Statistical Characterization

In this section, we evaluate the PDF and Cumulative distribution function (CDF) of the end-to-end SNR of the considered system for the DF relaying protocol. Utilizing Eq. (1), the instantaneous SNR of S-R link can be written as follows:

$$\gamma_{SR} = h_{SR}^t \bar{\gamma}_{SR}. \quad (5)$$

In Eq. (5), when  $t=1$  indicates HD technique and  $t=2$  shows IM/DD technique. The average SNR of the S-R link is shown by  $\bar{\gamma}_{SR}$ . Further, the CDF of  $\gamma_{SR}$  can be computed as

$$F_{\gamma_{SR}}(\gamma) = \Pr[h_{SR}^t \bar{\gamma}_{SR} \leq \gamma] = \int_0^{\left(\frac{\gamma}{\bar{\gamma}_{SR}}\right)^{\frac{1}{t}}} f_{h_{SR}}(h) dh. \quad (6)$$

Then, utilizing Eq. (6) the series-based expression for CDF of  $\gamma_{SR}$  can be calculated as

$$F_{\gamma_{SR}}(\gamma) = \frac{w_{zeq}^2 \sqrt{\eta}}{4\sigma_2^2} \sum_{n=0}^{\infty} \left( \frac{(\eta-1)w_{zeq}^2}{8\sigma_2^2} \right)^{2n} \left( \frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2} \right)^{-(2n+1)} \times \frac{1}{2^{2n}(n!)^{2n}} \Gamma\left(2n+1, \left( \frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2} \right) \ln \left( \frac{h_a A_0}{\gamma \bar{\gamma}_{SR}^{1/t}} \right) \right). \quad (7)$$

In Eq. (7),  $\Gamma(\cdot)$  represents the upper incomplete Gamma function. Likewise, using Eq. (3), the PDF of  $\gamma_{SR}$  is computed as

$$f_{\gamma_{SR}}(\gamma) = \frac{\gamma^{\left(\frac{1}{t}-1\right)}}{t(\bar{\gamma}_{SR})^{\frac{1}{t}}} f_{h_{SR}}\left(\left(\frac{\gamma}{\bar{\gamma}_{SR}}\right)^{\frac{1}{t}}\right). \quad (8)$$

In case of the R-U link, the RF signal is transmitted to U who is situated on the earth, it suffers from multipath fading due to various types of obstacles present on the ground. Consequently, the signal gets severely faded. Herein, to model the multipath fading behavior of the RF link, we exploit the Nakagami- $m$  distribution which aptly characterizes the multipath behavior of the RF signal. Thus, the PDF and CDF of the instantaneous SNR of the R-U link can be expressed as, respectively

$$f_{\gamma_{RU}}(\gamma) = \frac{1}{\Gamma(m)} \left( \frac{m}{\bar{\gamma}_{RU}} \right)^m \gamma^{m-1} \exp\left(-\frac{m}{\bar{\gamma}_{RU}} \gamma\right), \quad (9)$$

$$F_{\gamma_{RU}}(\gamma) = 1 - \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{m}{\bar{\gamma}_{RU}} \gamma\right). \quad (10)$$

In Eq. (10),  $\bar{\gamma}_{RU} = \Omega \bar{\gamma}_{RU}$ ,  $m$  shows fading severity, and  $\Omega$  average power. Here,  $\Gamma(\cdot)$  symbolizes the complete gamma function.

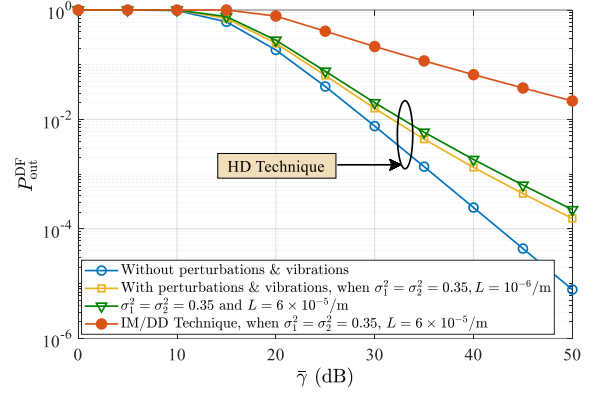


Fig. 2. Outage probability versus average SNR for different system parameters.

The end-to-end SNR of the considered system for the DF relay is  $\gamma_{DF} = \min(\gamma_{SR}, \gamma_{RU})$ . Thus, the end-to-end CDF can be expressed as

$$F_{\gamma_{DF}}(\gamma) = \Pr[\min(\gamma_{SR}, \gamma_{RU}) < \gamma], \\ = F_{\gamma_{SR}}(\gamma) + F_{\gamma_{RU}}(\gamma) - F_{\gamma_{SR}}(\gamma) \cdot F_{\gamma_{RU}}(\gamma). \quad (11)$$

## IV. Outage Probability Analysis

In this section, we derive the novel series-based expression of the outage probability ( $P_{out}^{DF}$ ) of the considered system model. It is the probability of communication link failure when end-to-end instantaneous SNR falls below a target SNR value  $\gamma_{th}$ . Thus, the outage probability of the considered system can be obtained by setting  $\gamma = \gamma_{th}$  in Eq. (11) as follows:

$$P_{out}^{DF} = \frac{w_{zeq}^2 \sqrt{\eta}}{4\sigma_2^2} \sum_{n=0}^{\infty} \left( \frac{(\eta-1)w_{zeq}^2}{8\sigma_2^2} \right)^{2n} \left( \frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2} \right)^{-(2n+1)} \frac{1}{2^{2n}(n!)^{2n}} \Gamma(2n+1, \left( \frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2} \right) \ln \left( \frac{h_a A_0}{(\gamma_{th}/\bar{\gamma}_{SR})^{1/t}} \right)) + \left( 1 - \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{m}{\bar{\gamma}_{RU}} \gamma_{th}\right) \right) - \left( \frac{w_{zeq}^2 \sqrt{\eta}}{4\sigma_2^2} \sum_{n=0}^{\infty} \left( \frac{(\eta-1)w_{zeq}^2}{8\sigma_2^2} \right)^{2n} \left( \frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2} \right)^{-(2n+1)} \frac{1}{2^{2n}(n!)^{2n}} \Gamma(2n+1, \left( \frac{(1+\eta)w_{zeq}^2}{8\sigma_2^2} \right) \ln \left( \frac{h_a A_0}{(\gamma_{th}/\bar{\gamma}_{SR})^{1/t}} \right)) \right) \left( 1 - \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{m}{\bar{\gamma}_{RU}} \gamma_{th}\right) \right). \quad (12)$$

## V. Numerical Result

Figure 2 shows the impact of different system parameters on the outage probability when  $\bar{\gamma}_{SR} = \bar{\gamma}_{RU} = \bar{\gamma}$ . It is observed that outage probability performance worsens with the presence of perturbations and vibrations, higher values of  $L$ , and when IM/DD technique is utilized.

## Acknowledgement

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## References

- [1] G. Wang, *et. al.*, "Free Space Optical Communication for Inter-Satellite Link: Architecture, Potentials and Trends," *IEEE Commun. Mag.*, vol. 62, no. 3, pp. 110-116, Mar. 2024,