Analytical Design of an LC Double-Notch Purcell Filter for Superconducting Quantum Circuits

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***Abstract*—Fast and reliable readout of superconducting qubits often causes energy to leak into the $50\,\Omega$ line, which is known as the Purcell effect, that shortens qubit lifetime. To address this issue, this paper has proposed a compact lumped LC filter that introduces two notches around the qubit frequency. These notches make the line appear nearly open to the qubit while keeping the readout band wide and flat. Using simple frequency sweeps, the article evaluate transmission, input admittance, and a Purcell lifetime proxy. The results show suppression of decay, robustness to component tolerances, and tuning with a small trim capacitor.**

***Index Terms*—cQED, Purcell filter, resonator networks, superconducting circuits.**

I. Introduction

Superconductor electronics uses passive and active superconducting components and resistors for developing functional circuits and systems. Circuit quantum electrodynamics (QED) has shown it is a potential and promising platform for the investigation of light-matter interactions, quantum information processing, and quantum simulation [1], [2], [3].

A Purcell filter is a component in superconducting qubit systems, designed to decrease the Purcell effect, an unwanted phenomenon where a qubit's energy decays into its readout resonator and then into the environment[4], [5], [6].

The design of the compact five-element LC filter that puts two transmission notches around the qubit frequency so the line looks "open" to the qubit, while the readout band stays wide and flat.

The rest of this paper is structured as follows. Section II explains the design methodology, covering the derivation of the system matrices, the frequency-domain analysis, and the formulation of the Lagrangian and Hamiltonian. Section III details the component parameters and their chosen values. Section IV presents the results. Finally, section V is the conclusion.

II. Design Method

To model the filter, a series capacitor contributes an impedance that decreases as frequency increases, while each shunt branch is made from a small LC series circuit to ground. The admittance of such a branch rises sharply near its own resonance, shorting to ground at that frequency and creating a deep notch in transmission. The impedance of a series capacitor Z_C is as follows:

$$Z_C(C) = \frac{1}{\mathrm{j}\,\omega C}\,, \tag{1}$$

where C is the capacitance and ω is the angular frequency, which Z_C decreases as frequency increases. For shunt series LC admittances (series branch to ground), we define Y_{sLC}:

$$Y_{\mathrm{sLC}}(L, C) = \frac{1}{\mathrm{j}\,\omega L + \dfrac{1}{\mathrm{j}\,\omega C}} = \frac{\mathrm{j}\,\omega C}{1 - \omega^2 LC}\,, \tag{2}$$

where L is inductance. Near resonance, the admittance spikes, meaning the branch strongly couples to ground and creating a notch. Each shunt branch shorts to ground at its series–resonance, where ω_z is the exact frequency of the notch:

$$\omega_z = \frac{1}{\sqrt{LC}}. \tag{3}$$

At this frequency, the LC branch cancels reactances and shorts to ground, producing the transmission zero.

A. Network matrices

In modeling the LC double-notch Purcell filter, each series capacitor and shunt LC branch is represented by an ABCD matrix, and cascading them yields the overall transfer matrix:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}. \tag{4}$$

While the scattering parameters $S_{21}(\omega)$ and $S_{11}(\omega)$ can be derived from this matrix, the most important quantity for qubit protection is the input admittance seen by the qubit:

$$Y_{\text{in}}(\omega) = \frac{\mathbf{A}Y_0 + \mathbf{C}}{\mathbf{B}Y_0 + \mathbf{D}}, \tag{5}$$

where $Y_0 = \frac{1}{\mathbf{Z}_0}$.

B. Lagrangian matrices

The kinetic energy ($\mathcal{T}$), comes from the capacitors ($\mathcal{T} \propto C\,\dot{\phi}^2$) and the potential energy ($\mathcal{V}$) from the inductors ($\mathcal{V} \propto \phi^2/L$). Together they form a quadratic Lagrangian. The kinetic (capacitive) energy is:

$$\mathcal{T} = \tfrac{1}{2}C_c\,\dot{\phi}_A^2 + \tfrac{1}{2}C_2\,(\dot{\phi}_A - \dot{\phi}_B)^2 + \tfrac{1}{2}C_1\,\dot{\psi}_1^2 + \tfrac{1}{2}C_3\,\dot{\psi}_2^2, \tag{6}$$

And the potential (inductive) energy (series branches referenced to ground) expresses as:

$$\mathcal{V} = \tfrac{1}{2}\,\frac{(\phi_A - \psi_1)^2}{L_1} + \tfrac{1}{2}\,\frac{(\phi_B - \psi_2)^2}{L_2}, \tag{7}$$

So the Lagrangian matrices is as follows:

$$\mathcal{L} = \tfrac{1}{2}\,\dot{\boldsymbol{\varphi}}^T\,\boldsymbol{C}\,\dot{\boldsymbol{\varphi}} - \tfrac{1}{2}\,\boldsymbol{\varphi}^T\,\mathbf{L}^{-1}\boldsymbol{\varphi}, \tag{8}$$

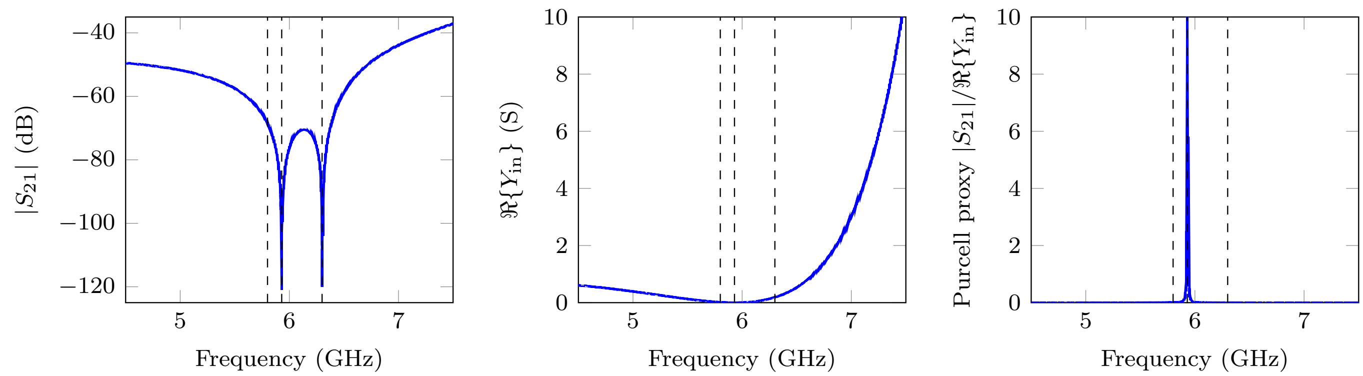

Figure 1. Result of the double-notch response and the corresponding Purcell metric, which forms two deep transmission notches at ∼5.93 and ∼6.30 GHz, that bracket the qubit at 5.80 GHz. In simulation, $|S_{21}|$ drops below -100 dB at each notch and remains more than 40dB down across a few hundred megahertz window around the qubit, while the readout band stays wide with ≤ 2 dB ripple.

Table I
COMPONENT VALUES OF THE DESIGNED FIVE ELEMENT LC DOUBLE-NOTCH PURCELL FILTER

Component	Value
Input coupling capacitor C_c	$8fF$
Inductor L_1	$8nH$
Capacitor C_1	$90fF$
Inductor L_2	$7.5nH$
Capacitor C_3	$85fF$
Series capacitor C_2	$40fF$

where the generalized momenta is $\boldsymbol{q} = \partial L/\partial\dot{\boldsymbol{\varphi}} = C\,\dot{\boldsymbol{\varphi}}$. Here, the dot ($\dot{}$) denotes the time derivative of the corresponding variables.

C. Hamiltonian of the network

The derivation leads to a multi-mode Rabi Hamiltonian (10), where the qubit couples to the resonator modes with strength g_k.

$$\hat{\mathcal{H}} = \sum_k \hbar\omega_k\, \boldsymbol{a}_{\boldsymbol{k}}^{\dagger}\boldsymbol{a}_{\boldsymbol{k}} + \frac{\hbar\omega_q}{2}\,\boldsymbol{\sigma}_{\mathrm{z}} + \sum_k \hbar g_k\big(\boldsymbol{a}_{\boldsymbol{k}} + \boldsymbol{a}_{\boldsymbol{k}}^{\dagger}\big)\,\boldsymbol{\sigma}_{\mathrm{x}}. \quad (9)$$

III. RESULTS

The presented LC Purcell filter forms two deep transmission notches at ∼5.93 and ∼6.30 GHz, as in Fig. 1, that bracket the qubit at 5.80 GHz. In simulation, $|S_{21}|$ drops below -100 dB at each notch and remains more than 40dB down across a few hundred megahertz window around the qubit, while the readout band stays wide with lower than 2 dB ripple. The input admittance seen by the qubit collapses in this valley (visible as a large peak in the Purcell lifetime proxy), and a simple $\pm 2\%$ tweak of the middle capacitor C_2 restores pass-band flatness after small parameter shifts with little change in the notch locations. Relative to a single-pole baseline (one shunt LC plus a coupling capacitor), the LC delivers a much wider stop band and more than 10 times improvement in the Purcell proxy at the same readout coupling, and it is far less sensitive to frequency drift because the qubit sits between two notches rather than on top of one.

IV. CONCLUSION

This paper has demonstrated a compact LC Purcell filter that creates two transmission notches around the qubit frequency, suppressing decay while preserving readout band. The design can reach passband flatness smoothly, and shows an improved magnitude, compared to a single notch baseline, and Purcell protection.

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