Fidelity Analysis for Dephasing Channels in Two-Qubit Systems

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***Abstract*—Dephasing noise, as a dominant source of decoherence in quantum systems, directly limits the reliability of entanglement-based and coherence-based protocols. This paper analyzes the fidelity of two-qubit states under independent, correlated, and partially correlated dephasing channels. The results highlight the critical role of noise correlations in preserving quantum coherence and provide insights for designing resilient quantum communication and computation systems.**

I. Introduction

Quantum information processing relies on superposition and entanglement, but unavoidable interactions with the environment cause decoherence, degrading correlations and hindering scalable technologies such as quantum communication, computation, and distributed networks [1]. Among noise processes, dephasing is especially critical, as it disrupts phase coherence [2], erodes entanglement [3], and threatens entanglement-based protocols like quantum anonymous communication [4] and quantum key distribution [5]. Dephasing can be independent, when qubits couple to separate environments; correlated, when they share common fluctuations; or partially correlated, which combines both local and collective effects.

To characterize quantum resource resilience, we use the Kraus-operator formalism to analyze two-qubit dephasing across three regimes, deriving analytical expressions for the fidelity of Bell and maximally coherent states. The remainder of this paper is organized as follows. Sec II introduces the dephasing noise models, including independent, correlated, and partially correlated channels. Sec. III presents fidelity analysis and graphical results showing correlation effects on robustness. Finally, Sec. IV consists of our concluding remarks.

II. Dephasing Noise Models

A. Independent Dephasing Channel

When dephasing is independent, each qubit interacts with a distinct environment, leading to uncorrelated phase noise. In the symmetric scenario with identical probability p, the channel is defined as

$$\begin{aligned}\mathcal{E}_{\text{ind}}(\rho) = (1-p)^2 (I\otimes I)\,\rho\,(I\otimes I) \\ + p(1-p)\,(Z\otimes I)\,\rho\,(Z\otimes I) \\ + p(1-p)\,(I\otimes Z)\,\rho\,(I\otimes Z) \\ + p^2 (Z\otimes Z)\,\rho\,(Z\otimes Z),\end{aligned} \tag{1}$$

which represents fully local decoherence, where each qubit loses its phase coherence independently.

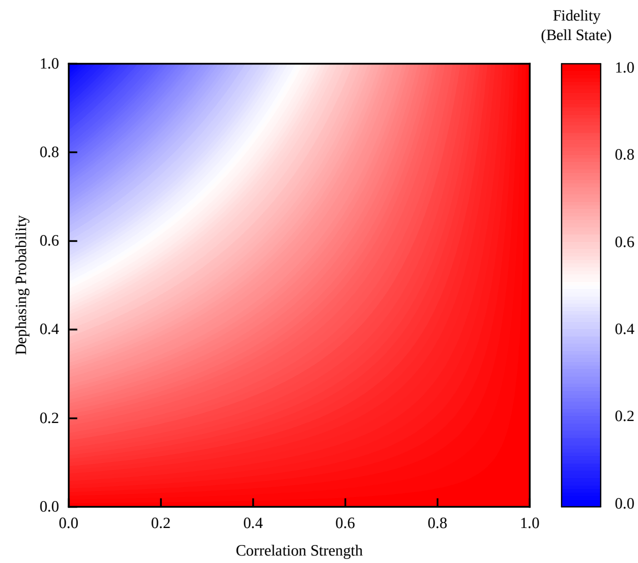

Fig. 1. Fidelity of maximally entangled two-qubit states under partially correlated dephasing channels.

B. Correlated Dephasing Channel

When both qubits are subject to the same phase fluctuation arising from a common environment, the corresponding correlated dephasing map is expressed as

$$\begin{aligned}\mathcal{E}_{\text{corr}}(\rho) = (1-p)\,(I\otimes I)\rho(I\otimes I) \\ + p\,(Z\otimes Z)\rho(Z\otimes Z),\end{aligned} \tag{2}$$

where p denotes the probability of a collective phase flip acting on both qubits simultaneously.

C. Partially Correlated Dephasing Channel

With local and collective phase errors acting simultaneously, the partially correlated dephasing channel is expressed as

$$\begin{aligned}\mathcal{E}_{\text{par}}(\rho) = (1-p)\,(I\otimes I)\rho(I\otimes I) \\ + \frac{p(1-\kappa)}{2}\,(Z\otimes I)\rho(Z\otimes I) \\ + \frac{p(1-\kappa)}{2}\,(I\otimes Z)\rho(I\otimes Z) \\ + p\kappa\,(Z\otimes Z)\rho(Z\otimes Z),\end{aligned} \tag{3}$$

where $p \in [0,1]$ is the overall dephasing probability and $\kappa \in [0,1]$ quantifies the correlation strength between the two

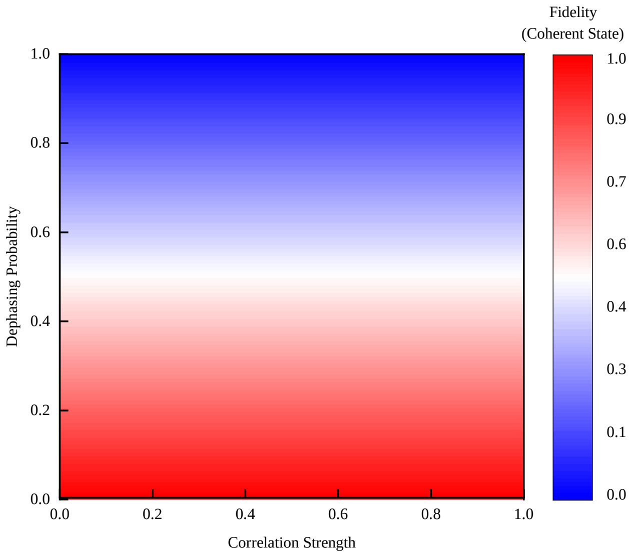

Fig. 2. Fidelity of maximally coherent two-qubit states under partially correlated dephasing channels.

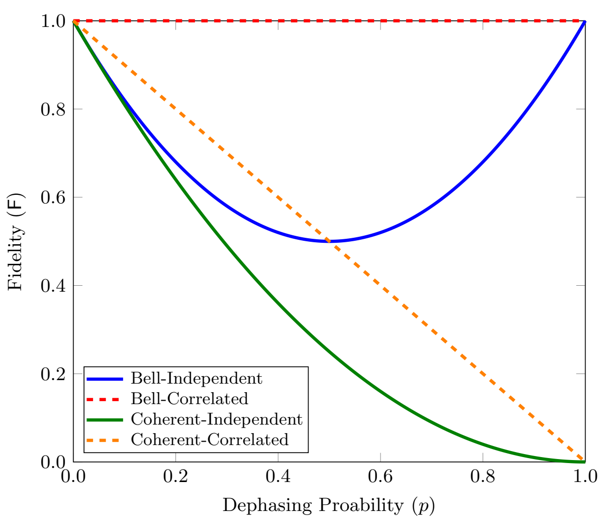

Fig. 3. Fidelity of maximally entangled and maximally coherent two-qubit states under independent and correlated dephasing channels.

qubits. Specifically, $\kappa = 0$ corresponds to fully independent local dephasing, while $\kappa = 1$ represents perfectly correlated (collective) phase noise.

III. Fidelity Analysis and Results

To quantify the impact of dephasing noise on quantum coherence and entanglement, we evaluate the state fidelity [6] between the noisy output ρ' and the original pure state $|\psi\rangle$ as

$$\mathsf{F}(\rho', |\psi\rangle) = \langle\psi| \rho' |\psi\rangle , \qquad (4)$$

which ranges from 0 (complete decoherence) to 1 (perfect preservation). We consider two representative two-qubit states: the maximally entangled Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, and the maximally coherent state $|\psi_c\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$.

A. Results

As shown in Fig. 1 and Fig. 2, fidelity heatmaps reveal the combined effect of p and κ under partially correlated dephasing. The Bell state gains robustness as κ increases (approaching the correlated limit), whereas the coherent state depends mainly on p and is only weakly sensitive to κ. A comparison of fidelity for independent and correlated channels is presented in Fig. 3, detailing its dependence on dephasing probability. The Bell state retains perfect fidelity under correlated dephasing, while independent noise leads to quadratic decay. The coherent state, however, shows linear degradation for correlated dephasing and faster quadratic decay for independent noise.

IV. Conclusions

In this study, we analyzed the effects of independent, correlated, and partially correlated dephasing on the fidelity of two-qubit states. Our results show that entangled states exhibit strong robustness in correlated environments, revealing the presence of a decoherence-free subspace. In contrast, non-entangled coherent states remain susceptible to decoherence regardless of the correlation strength. These findings highlight the crucial role of collective noise dynamics in preserving quantum coherence and establish a theoretical foundation for the design of resilient quantum communication and computation protocols in realistic, correlated environments.

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