A Study of Hook Error Suppression on the Heavy Hexagon Lattice

Shengyue Heng, Yujin Kang, Jun Heo *

Korea Univ., Korea Univ., *Korea Univ.

shengyue98@korea.ac.kr, yujin20@korea.ac.kr, *junheo@korea.ac.kr

Heavy Hexagon 구조에서 Hook 오류 방지 방법에 대한 연구

형서은, 강유진, 허준*
고려대학교, 고려대학교, *고려대학교

Abstract

This study focuses on quantum hardware architectures that different from the conventional square lattice, such as IBM's heavy-hexagon lattice, where standard quantum error correction codes cannot be directly applied. Due to limited connectivity, each qubit connects with only two or three neighboring qubits via additional qubits such as bridge or flag qubits. Using the Surface Code-17 as an example, we modify the order of CNOT gates in the stabilizer measurement circuit to avoid hook errors. Simulation results confirm that our proposed method effectively prevents error propagation in such lattice.

Ⅰ. Introduction

Quantum computers are highly susceptible to errors caused by environmental noise, which significantly limit reliable quantum computation [1]. To address this, quantum error correction (QEC) codes, including the surface code [2], have been proposed to ensure fault-tolerant quantum operations.

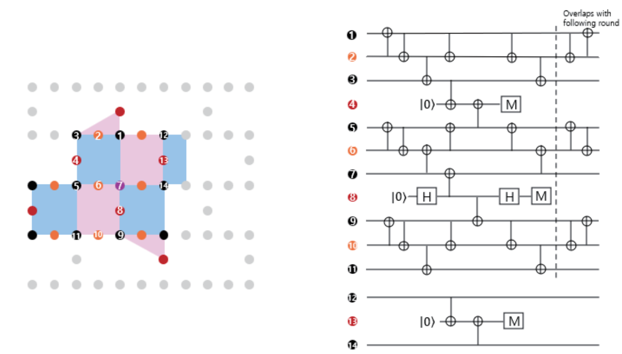
Previous studies have focused on mitigating hook errors in the surface code on a 2D square lattice with next-nearest-neighbor connectivity of 4 [3]. However, this configuration is incompatible with IBM quantum devices, which adopt a heavy-hexagonal lattice with limited connectivity {2, 3} to reduce crosstalk and gate errors [4-5]. The low connectivity complicates hook error suppression and requires extra elements such as swap gates or flag qubits, leading to increased error propagation.

In this paper, we change the sequence of CNOT gates in the stabilizer measurement circuit on the heavy-hexagon lattice and show that hook errors can be avoided through simulation.

Ⅱ. SC-17 on the heavy-hexagon lattice

1. Previous work

In IBM quantum computers, physical qubits are arranged on the vertices and edges of a hexagonal lattice. Following [5], we adopt the embedding and qubit arrangement of the rotated surface code (SC-17) in our implementation shown in Fig.1.



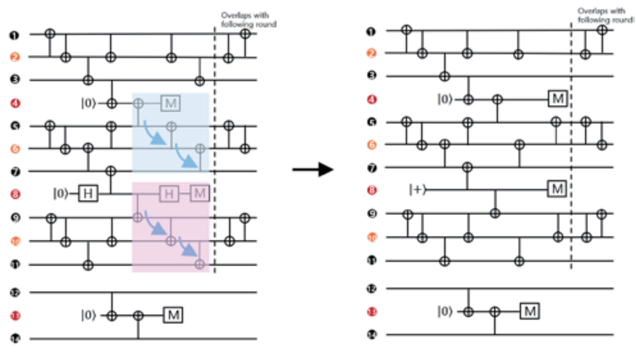
(a) (b)

Fig.1 The SWAP-based layout of SC-17 on the heavy-hexagon lattice (a) and a sub-round syndrome extraction circuit measures weight-four and weight-two stabilizers including those at the side boundaries (b)

2. Propose method

A hook error occurs when a single physical error propagates to two data qubits aligned with a logical operator.

As shown in Fig. 2(a), X(Z)-type stabilizer measurement circuits can copy X(Z) errors onto data qubits. Copying to four qubits applies the stabilizer, while copying to three represents a single error modulo a stabilizer. When two aligned qubits are affected, a hook error arises, allowing the logical operator to form with only half as many physical errors as the code distance implies. Fig. 2(b) changes the order of the second and the third CNOT in both blue and pink area to prevent the error propagation.



(a) (b)

Fig.2 The original circuit (a) annotated with hook error locations and the modified circuit (b)

3. Simulation results

Firstly, we implemented standard depolarizing circuit-level noise, which are shown in table 1 below [6].

Qubit operation	Error	Probability
CNOT	Each two-qubit gate is followed by an element of $\{X,Y,Z,I\}^{\otimes 2}\backslash\{II\}$	p
Hadamard	each single-qubit gate is followed by a Pauli X, Y, or Z error	p
Measurement	Report incorrect result and project to orthogonal eigenstate	p

Table.1 Standard depolarizing noise applied to the gates in our circuit

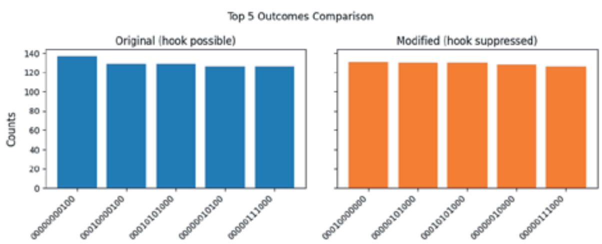


Fig.3 Simulation for our proposed CNOT order

From the figure 3, we can see that the effect of hook errors is on the comparison between original circuit and modified circuit for SC-17. For each circuit, we extracted the five qubit sequences with the highest counts, ordered according to the qubit numbering (1–11) in Fig. 2. As shown in the left figure, instances with more than two data qubits in the "1" state are observed, while the right figure shows no such occurrences. These results demonstrate that the modified circuit successfully suppresses hook errors and prevents error propagation.

Ⅲ. Conclusion

In this paper, we introduced a new CNOT order of the surface code on the heavy-hexagon lattice and confirmed that our proposed method can prevent from error propagation in such circuit.

The proposed approach identified and mitigated potential hook errors by reordering CNOT gates, effectively preventing error propagation. Simulation results showed that errors affecting more than two data qubits are suppressed in the modified circuit.

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