Dual-User STAR-RIS-Assisted Uplink NOMA: Fairness Optimization

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Abstract

This paper investigates the use of simultaneously transmitting and reflecting intelligent surface (STAR-RIS) to enhance uplink non-orthogonal multiple access (NOMA) for two users. We propose a joint optimization of power allocation and STAR-RIS beamfomrming maximizing the rate fairness among users without resorting to alternating optimization.

Ⅰ. Introduction

The concept of simultaneously transmitting and reflecting intelligent surface (STAR-RIS) have emerged as a key technology for providing 360° coverage through simultaneous transmission and reflection [1]. When combined with non-orthogonal multiple access (NOMA), which improves spectral efficiency and connectivity, this combination shows strong potential for future efficient wireless systems [2]. Previous studies have investigated minimum-rate maximization in RIS-assisted NOMA [3], and our recent work in [4] presented a general multi-user STAR-RIS framework. In this paper we focus on the dual-user case and propose simplified second-order cone program (SOCP) formulation that achieves max-min rate fairness with reduced computational complexity compared to the general multi-user case in [4].

Ⅱ. System Model and Problem Formulation

We consider an uplink NOMA system enhanced by a STAR-RIS, where a single-antenna base station (BS) communicates with two single-antenna users, through STAR-RIS without direct link between users and BS. The STAR-RIS operates in energy splitting mode, with N elements handling both transmission and reflection. For user $q\in\{t,r\}$ the transmission/reflection coefficients are given as:

$$\boldsymbol{\theta}_q = (\sqrt{\beta_{q,1}}e^{j\phi_{q,1}},\ \beta_{q,2}e^{j\phi_{q,2}},\cdots,\sqrt{\beta_{q,n}}e^{j\phi_{q,n}})^T \quad (1)$$

with t and r denoting transmission and reflection spaces, respectively. The channels between STAR-RIS and BS is denoted by $\boldsymbol{g}\in\mathbb{C}^{N\times 1}$ and that between STAR-RIS and each user q is denoted by $\boldsymbol{f}_q\in\mathbb{C}^{N\times 1}$. For the uplink NOMA, the received signal at the BS is expressed as:

$$y = \sqrt{p_t}\boldsymbol{h}_t^T\boldsymbol{\theta}_t s_t + \sqrt{p_r}\boldsymbol{h}_r^T\boldsymbol{\theta}_r s_r + n \quad (2)$$

where $\boldsymbol{h}_q = \boldsymbol{g}\circ\boldsymbol{f}_q$, p_q, s_q are the cascaded channel, transmit power and symbol of user q, respectively, with $E\left[|s_q|^2\right]=1$ and $n\sim\mathbb{CN}(0,\sigma^2)$ representing the additive white Gaussian noise at the BS.

We aim to maximize the minimum rate of the two users by jointly optimizing the transmission/reflection coefficients and power allocation as:

$$\max_{\boldsymbol{\Theta},\boldsymbol{p},\boldsymbol{\pi}} \min\left(\log_2\left(1+\frac{p_t\left|\boldsymbol{h}_t^T\boldsymbol{\theta}_t\right|^2}{\pi_r p_r\left|\boldsymbol{h}_t^T\boldsymbol{\theta}_r\right|^2+\sigma^2}\right),\log_2\left(1+\frac{p_r\left|\boldsymbol{h}_2^T\boldsymbol{\theta}_r\right|^2}{\pi_t p_t\left|\boldsymbol{h}_1^T\boldsymbol{\theta}_t\right|^2+\sigma^2}\right)\right) \quad (3a)$$

$$\text{s.t.}\quad |\theta_{tn}|^2+|\theta_{rn}|^2\le 1,\ n\in\mathrm{N}, \quad (3b)$$

$$0\le p_q\le P_q^{\max},\ q\in\{t,r\} \quad (3c)$$

$$\pi\in\Pi_q \quad (3d)$$

III. Optimization Approach

For a given successive interference cancellation (SIC) order and optimal phase shift $\phi_{q,n}=-\angle h_{q,n}$ along with transmission/reflection amplitude $\boldsymbol{B}=[\boldsymbol{\beta}_t,\boldsymbol{\beta}_r]$, and $\zeta_{qn}=\frac{|h_{qn}|}{\sigma}$ with $\boldsymbol{\zeta_q}=[\zeta_{q1},\zeta_{q2},\cdots,\zeta_{qN}]^T$ and by utilizing the monotonic increasing property of the log function, the problem can be reformulated as:

$$\max_{\boldsymbol{B},\boldsymbol{p}} \min\left(\frac{\sqrt{p_t}\zeta_t^T\boldsymbol{\beta_t}}{\sqrt{p_r(\zeta_r^T\boldsymbol{\beta_r})^2+1}},\sqrt{p_r}\zeta_r^T\boldsymbol{\beta_r}\right) \quad (4a)$$

$$\text{s.t.}\quad \beta_{\mathrm{tn}}^2+\beta_{\mathrm{rn}}^2\le 1,\ \beta_{\mathrm{tn}}\ge 0,\ \beta_{\mathrm{rn}}\ge 0,\ \mathrm{n}\in\mathrm{N} \quad (4b)$$

$$0\le p_q\le P_q^{\max},\ q\in\{t,r\} \quad (4c)$$

By defining $\boldsymbol{z}_q=\sqrt{p_q}\boldsymbol{\beta}_q$ with $\boldsymbol{Z}=[\boldsymbol{z}_t,\boldsymbol{z}_r]$ and $\widetilde{\chi}_1\le\min(f_1,f_2)$, thus the problem can be cast as SOCP and given as:

$$\max_{\boldsymbol{B},\boldsymbol{Z},\widetilde{\chi}_1} \widetilde{\chi}_1 \quad (5)$$

$$\text{s.t.}\quad \widetilde{\chi}_1\left\|[\boldsymbol{\zeta}_\mathbf{r}^\mathrm{T}\boldsymbol{z}_\mathbf{r},1]\right\|_2\le\boldsymbol{\zeta}_\mathbf{t}^\mathrm{T}\boldsymbol{z}_\mathbf{t},\ \widetilde{\chi}_1\le\boldsymbol{\zeta}_\mathbf{r}^\mathrm{T}\boldsymbol{z}_\mathbf{r},$$

$$\boldsymbol{z_q}\le\sqrt{P_q^{\max}}\,\boldsymbol{\beta}_q,\ q\in\{t,r\}$$

$$\left\|[\beta_{tn},\beta_{rn}]\right\|_2\le 1,\quad n\in\boldsymbol{N}$$

The optimal value $\widetilde{\chi}_1^*$ is then obtained by solving a sequence of feasibility problems.

III. Results and Discussion

For our simulations, we adopt a setup similar to [4], including the BS and STAR-RIS deployment, user distribution, and propagation model.

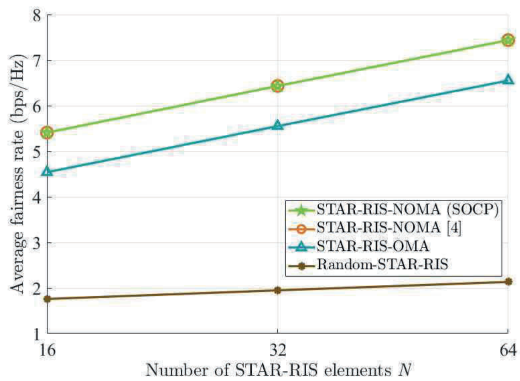

Fig.1. Achievable fairness rate vs number of STAR-RIS elements

We compare our low-complexity SOCP-based STAR-RIS NOMA with the reference STAR-RIS NOMA [4], orthogonal multiple access (OMA), and a random phase‑amplitude STAR-RIS baseline. Our method matches [4] at lower complexity, and both NOMA schemes consistently exceed OMA and the random baseline for all N.

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