# A Concise Review of Channel Estimation Approaches in Orthogonal Time Frequency Space (OTFS) Systems

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## Abstract

This paper discusses the challenges that Orthogonal Time Frequency Space (OTFS) faces in channel estimation, including fractional Doppler effects, pilot design, continuous Doppler spread channels, and multi-user multiple-input multiple-output (MU-MIMO) systems. The article summarizes current delay-Doppler (DD) domain channel estimation methods, aiming to promote the application of OTFS in 6G networks.

## I. Introduction

Each generation of wireless networks introduces a new waveform based on specific use case needs. For 4G and 5G, Orthogonal Frequency-Division Multiplexing (OFDM) has been essential, but its limitations in high-mobility 6G networks require alternatives like Orthogonal Time Frequency Space (OTFS). OTFS is modulated in the delay-Doppler (DD) domain, addresses these challenges but also presents technical difficulties that are being studied and resolved. Accurate channel estimation (CE) is crucial for robust OTFS demodulation. This paper introduces system models, addresses current CE challenges, reviews representative works, and discusses their performance and complexity.

## II. System Model

### A. OFTS Modulation & Channel Model

Consider a set of $N \times M$ data symbols $\{X_{DD}[k,l]\}$ that are modulated in the DD domain, where $l \in [0,M)$ and $k \in [0,N)$ are delay and Doppler indices, respectively. Let $h_w(\nu,\tau)$ denote a channel impulse response with a total number of *P* paths, which is represented as

$$h_w(\nu,\tau) = \sum_{i=1}^{P} h_i e^{j2\pi\nu_i\tau_i} G(\nu,\nu_i) F(\tau,\tau_i),$$

where $F[\tau,\tau_i]$ and $G[\nu,\nu_i]$ are delay and Doppler sampling functions with delay tap $\tau_i$ and Doppler tap $\nu_i$, respectively, which are given as

$$F(\tau,\tau_i) \square \sum_{m'=0}^{M-1} e^{j2\pi(\tau-\tau_i)m'\Delta f}, \tag{1}$$

$$G(\nu,\nu_i) \square \sum_{n'=0}^{N-1} e^{-j2\pi(\nu-\nu_i)n'T}. \tag{2}$$

Let $l_{\tau_i}$ and $k_{\nu_i}$ be the integers to denote the delay and Doppler taps, respectively. Then, delay tap $\tau_i$ and Doppler tap $\nu_i$ can be represented in their fractional indices as follows:

$$\tau_i = \frac{l_{\tau_i} + \varsigma_{\tau_i}}{M\Delta f} \quad \text{and} \quad \nu_i = \frac{k_{\nu_i} + \kappa_{\nu_i}}{NT}$$

where $\varsigma_{\tau_i} \in (-0.5, 0.5]$ and $\kappa_{\nu_i} \in (-0.5, 0.5]$. Using an ideal transmit and receive pulses, i.e., biorthogonal waveforms, the received DD-domain signal $Y_{DD}[k,l]$ is now given as

$$Y_{DD}[k,l] = \frac{1}{NM} \sum_{k'=0}^{N-1} \sum_{l'=0}^{M-1} X_{DD}[k',l'] h_w[k-k', l-l'] \tag{3}$$

where $h_w[k,l]$ is the sample of impulse response function $h_w(\nu,\tau)$.

### B. Embedded Pilot-based Channel Estimation

Taking the integer delay and Doppler case for example, we first choose arbitrary pilot indices, $l_p$ & $k_p$, such that $0 \le l_p \le M-1$ and $0 \le k_p \le N-1$. We choose $0 \le l_p - l_\tau \le l_p \le l_p + l_\tau \le M-1$ and $0 \le k_p - 2k_\nu \le k_p \le k_p + 2k_\nu \le N-1$ as the CE region, where $l_\tau$ and $k_\nu$ are the maximum delay and Doppler values, respectively.

We arrange the pilot symbol, guard symbols, and data symbols in the delay–Doppler grid as

$$x[k,l] = \begin{cases} x_p, \ k = k_p, l = l_p, \\ 0, \ k_p - 2k_\nu \le k \le k_p + 2k_\nu, l_p - l_\tau \le l \le l_p + 2l_{\tau,} \\ x_{\text{data}}, \ \text{otherwise.} \end{cases} \tag{4}$$

For the CE region,

$$y[k,l] = \hat{h}[k-k_p, l-l_p] x_p, \tag{5}$$

Then the channel is estimated as

$$\hat{h}[k-k_p, l-l_p] = \begin{cases} \dfrac{y[k,l]}{x_p}, \ |y[k,l]| \ge \text{Threshold}, \\ 0, \ |y[k,l]| < \text{Threshold}. \end{cases} \tag{6}$$

## III. Challenges and Present Methods

CE in the DD domain may seem straightforward at first glance, but upon closer examination, there are still many challenges that need to be addressed.

### A. Fractional Doppler and Other Interferences

Due to the finite bandwidth and frame duration, the corresponding delay and Doppler resolution of the DD grid are also limited. As a result, the continuous delays and Doppler shifts in the physical channel may not fall exactly on the integer grid points of the DD domain. This phenomenon is referred as fractional (off-grid) delay and Doppler. Due to $\varsigma_{\tau_i}$ and $\kappa_{\nu_i}$, sampling functions are non-zeroes for $l - l' \ne l_{\tau_i}$ and $k - k' \ne k_{\nu_i}$, which means the DD-domain channel is not that sparse. There is a wide range of opinions on this issue. Due to the additional complexity required to estimate fractional Doppler, some researchers believe it is not worth the effort; on the other hand, if OTFS is to be widely used for various environments and applications in 6G, since the velocity difference between scatterers might not always be so large, fractional Doppler is indeed inevitable. For this problem, the DD domain channel can be modeled as a sparse matrix with block structure characteristics, and then compressed sensing (CS) methods are used to formulate it as the following sparse signal recovery problem:

$$\min \|\boldsymbol{h}\|_0, \boldsymbol{y}_p = \boldsymbol{X}_p \boldsymbol{h} + \boldsymbol{w} \tag{7}$$

where $\boldsymbol{y}_p \in C^{MN\times 1}$ , $\boldsymbol{X}_p \in C^{MN\times MN}$ , $\boldsymbol{h} \in C^{MN\times 1}$ , $w$ is the additive noise. Orthogonal Matching Pursuit (OMP), Modified Subspace Pursuit (MSP) algorithm can be employed [1].

The aforementioned input-output relation (2) is under the ideal condition with biorthogonal waveforms. Under practical pulse shaping, there is a possibility of inter-carrier interference (ICI) and inter-symbol interference (ISI). To overcome these problems, another class of methods known as Sparse Bayesian Learning (SBL) is used. The SBL framework does not require knowledge of the level of sparsity. It introduces an appropriate prior distribution on the sparse channel to capture its inherent sparsity and estimate it. Even if the pilot overhead of the SBL method is the same as that of the OMP, it is known to have 3dB better NMSE estimation performance under the given conditions [2].

The main issue is still fractional Doppler. Some significant efforts have emerged to overcome it. The most representative work is the off-grid SBL CE methods [3]. In the 1D approach, the grid points of delay and Doppler, along with their corresponding off-grid components, are jointly estimated, implying that they are coupled within the algorithm. In contrast, the 2D method first decouples the delay and Doppler, employing a two-step approach to estimate the off-grid delay components, Doppler components, and channel coefficients. The complexity of the algorithm is approximately dominated by $\mathrm{O}({M_\tau}^2 {N_\nu}^2)$ for 1D and $\mathrm{O}({M_\tau}^2 N_\nu)$ for 2D algorithm, where $M_\tau$ and $N_\nu$ are the sizes of the virtual sampling grids in the delay and Doppler domain, respectively. It indicates an obvious trade-off between estimation accuracy and complexity.

Deep neural network (DNN) can be useful for fractional Doppler CE, which uses fully connected neural network to directly estimate channel parameters and then reconstruct channel matrix [4]. The complexity of it is mainly governed by the numbers of neurons. From the simulation results, it can be seen that the DNN method does not outperform the OMP algorithm in terms of NMSE performance. The network structure seems to be optimizable, reducing its complexity and improving its performance.

### *B. Pilot Design*

Traditional CE schemes transmit pilot pulses separately within a single OTFS frame. In spite of its low complexity, it has low spectral efficiency. Alternatively, an embedded pilot-aided CE method [5] has been proposed. As discussed in Section II, it requires that pilot symbols be surrounded by guard bands to prevent interference between the pilot and data symbols, followed by using hard threshold decisions in (6). This method improves spectral efficiency. Additionally, it will significantly increase the Peak-to-Average Power Ratio (PAPR) of the time-frequency (TF) domain waveform, posing challenges for hardware design. It is shown in [5] that under the given conditions, the single pilot SNR needs to reach 40-50 dB, and with data SNR of 16 dB, BER performance can be maintained at around $10^{-4}$ .

Superimposed pilot and data transmission scheme in [6] utilizes the entire frame for data transmission and overlays a single pilot symbol onto the data symbols as follows:

$$\tilde{x}[k,l] = \begin{cases} x_p + x_{\text{data}}[k_p, l_p], k = k_p, l = l_p, \\ x_{\text{data}}[k,l], \text{otherwise.} \end{cases} \tag{8}$$

Consequently, its spectral efficiency is further improved. Even if interference between data and pilot is inevitable, it has been shown that it has an NMSE performance that is close to that of the embedded pilot methods.

Furthermore, pseudo-random noise (PN)-based pilots in [7] can maintain a relatively low PAPR. Implemented in the TF-domain, it does not utilize the channel's doubly selective characteristics, hence it has a high implementation complexity, which changes with the sequence length. Meanwhile, BER performance improves with the sequence length, which implies that there is still a trade-off between CE error performance and complexity.

Up to now, for the sake of analytical convenience, most current research on OTFS receivers is widely based on embedded or superimposed pilots for further improvements in CE and symbol detection. We note that most works are based on these two methods for different scenarios and conditions.

### *C. Multi-user Multiple-Input Multiple-Output (MU-MIMO)*

In massive MIMO system, OTFS is challenging as orthogonal pilot signals are required to distinguish different antennas, incurring high pilot overhead. For traditional CE techniques in massive MIMO systems with $N_t$ transmit antennas, the pilot overhead is proportional to $N_t N_{\max} M_{\max}$ , where $M_{\max}$ and $N_{\max}$ denote the largest path delay and Doppler grids, respectively. Meanwhile, CS techniques can be used to estimate the channel with fewer pilot signals. Toward this end, the CE problem can be transformed into a delay-Doppler-Angular sparsity problem, which can be solved by some CS methods, including 3D-OMP, 3D-Newtonized OMP, and 3D- MSP. In 3D-OMP CE technique with the length $D$ of non-zero block along the angle dimension, the pilot overhead is proportional to $N_{\max} PD \log(N_g M_g N_t)$ where $M_g$ and $N_g$ are the length of guard intervals along delay and Doppler axis, respectively. We note that the logarithmic term arises from the nature of CS algorithms, which typically require a number of measurements proportional to the logarithm of the dimensionality of the problem space.

## IV. Discussions and Conclusions

Due to the favorable properties of the DD-domain channel, current CE methods for OTFS are mainly carried out in the DD domain. Compared to CE in the TF-domain, DD-domain CE has the advantages of lower complexity and reduced overhead. This paper provides an overview of the input-output relation in OTFS, summarizes some of the challenges, existing work, and limitations in the current DD-domain CE, with the hope of providing valuable references for the development of OTFS CE methods.

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