Performance Analysis of Variational Quantum Circuit: An Application to Beamforming Optimization

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***Abstract*—This paper investigates the performance of variational quantum circuits (VQC) across different circuit designs. The VQC is applied to optimize the precoder-combiner in multiple-input multiple-output (MIMO) systems. The results show that VQC with circuit 1 has a lower training loss, resulting in a higher achievable sum rate compared to VQC circuit 2.**

***Index Terms*—combiner, precoder, MIMO, variational quantum circuit.**

I. Introduction

Variational Quantum Circuit (VQC) is a technique that combines the principles of quantum computing with machine learning to potentially achieve lower computational complexity [1]. The VQC process typically involves three crucial steps: data encoding, weight encoding, and measurement [2]. Data encoding transforms classical information into quantum states, allowing the exploitation of quantum phenomena. Weight encoding maps the model's parameters onto quantum gates, enabling the learning algorithm to operate within the quantum framework. Finally, measurement extracts classical results from the quantum computations [3]. The effectiveness of VQCs is dependent on the design of the quantum circuit, as the arrangement and selection of quantum gates determine the efficiency and accuracy of the computations. This paper provides a performance comparison of two different circuit designs. In a specific case, different VQC circuits are employed to optimize the precoder-combiner in multiple-input multiple-output (MIMO) systems.

II. Variational Quantum Circuit

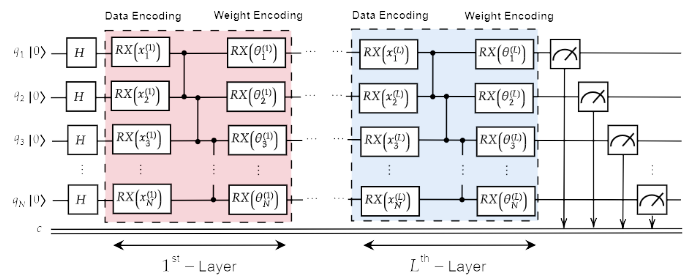
This study considers two different quantum circuits for the optimization task. The training process in quantum machine learning can be divided into pre-processing, feed-forward, and decoding, which can be explained as follows.

A. Pre-processing

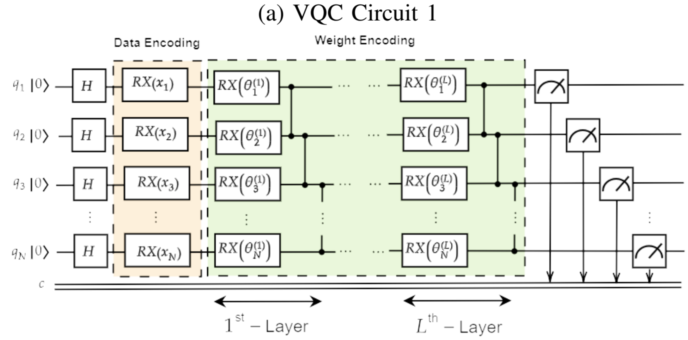
In the quantum system, the quantum bits (qubits) can be prepared in superposition states, which can be expressed as

$$\mathcal{U}^{[\text{VQC-1}],[\text{VQC-2}]}_{\text{pre-processing}} \triangleq \bigotimes_{n=1}^{N_{\text{qubits}}} \mathbb{H}\big(|x_n\rangle\big), \tag{1}$$

where $\mathbb{H}$ denotes the Hadamard gate for superposition representation.



(a) VQC Circuit 1



(b) VQC Circuit 2

Figure 1: The considered quantum circuit.

B. Feed-forward VQC Circuit 1

Afterward, the data is processed in the feedforward process, there is data encoding and weight encoding. VQC circuit 1 utilizes the quantum embedding techniques as described in [4], where classical data and weights are encoded into quantum states and repeated across multiple layers as depicted in Fig. 1a, which can be expressed as

$$\begin{aligned}\mathcal{U}_{\text{VQC-1}} \triangleq \Bigg(\bigotimes_{l=1}^{L_{\text{layer}}} \bigotimes_{n=1}^{N_{\text{qubit}}} \mathbf{RX}\Big(\theta_n^{[l]}\Big) \Bigg(\prod_{l=1}^{L_{\text{layer}}} \prod_{n=1}^{N_{\text{qubit}}} \mathbf{CX}\Big(q_n^{[l]} | q_{n-1}^{[l]}\Big) \\ \otimes \ldots \otimes \mathbf{CX}\Big(q_{N_{\text{qubit}}}^{(d)} | q_{N_{\text{qubit}}-1}^{(d)}\Big)\Bigg) \mathbf{RX}\Big(x_n^{[l]}\Big),\end{aligned} \tag{2}$$

where $\mathbf{RX}\Big(\theta_n^{[l]}\Big)$ and $\mathbf{RX}\Big(x_n^{[l]}\Big)$ denotes the weight encoding and data encoding of the $l-$th layer, respectively.

C. Feed-forward VQC Circuit 2

On the other hand, VQC Circuit 2 adopts a different approach, utilizing another type of circuit proposed in [5]. As illustrated in Fig. 1a, the data is encoded just once, and the weights are repeated as many times as there are layers, which can be expressed as

$$\mathcal{U}_{\text{VQC-2}} \triangleq \left(\prod_{l=1}^{L_{\text{layer}}} \prod_{n=1}^{N_{\text{qubit}}} \mathbf{CX}\left(q_n^{[l]} | q_{n-1}^{[l]}\right) \otimes \ldots \otimes \mathbf{CX}\left(q_{N_{\text{qubit}}}^{(d)} | q_{N_{\text{qubit}}-1}^{(d)}\right) \bigotimes_{l=1}^{L_{\text{layer}}} \bigotimes_{n=1}^{N_{\text{qubit}}} \mathbf{RX}\left(\theta_n^{[l]}\right) \right) \mathbf{RX}\left(x_n\right). \tag{3}$$

D. Decoding

Quantum measurement is carried out at the final layer of both VQC circuit 1 and circuit 2 to derive classical output values, expressed as

$$\mathbf{M}_{\text{VQC}}^{[1,2]} = \langle 0 | \, U_{\text{VQC}}^{[1,2]}(\Theta)^{\dagger} H U_{\text{VQC}}^{[1,2]}(\Theta) \, | 0 \rangle \, . \tag{4}$$

To minimize errors from noisy quantum computations, the measurement is repeated N_{shot} times, represented as

$$y = \frac{1}{N_{\text{shot}}} \sum_{n=1}^{N_{\text{shot}}} \mathbf{M}_{\text{VQC}}^{[1,2]}. \tag{5}$$

Since an unsupervised learning scheme is employed, the loss is computed as $\mathbb{L}_{\text{VQC}}^{[1,2]}(\theta) = -R_{\text{sum}}$. Using the calculated loss, the gradient is determined via the parameter-shift rule, given by

$$\nabla \mathbb{L}_{\text{VQC}}^{[1,2]}(\theta) = \frac{\mathbb{L}_{\text{VQC}}^{[1,2]}(\theta + \varpi) - \mathbb{L}_{\text{VQC}}^{[1,2]}(\theta - \varpi)}{2 \sinh(\varpi)}, \tag{6}$$

where $\varpi \in [0, \pi]$ represents the shifting phase. Finally, the weight parameters are updated according to $\theta^{[1,\,2]} = \theta^{[1,\,2]} - \alpha \nabla \mathbb{L}_{\text{VQC}}^{[1,2]}(\theta)$, with $\alpha \in (0, 1]$ denoting the learning rate.

III. Beamforming Optimization

In a specific application, the VQC is used to optimize beamforming to maximize the achievable sum rate. A precoder is employed at the base station (BS) to direct the signal to the intended users (UE). The optimal precoder is proposed in [6]. Meanwhile, at the receiver site, a combiner is employed to mitigate interference and extract the intended signal, as proposed in [7]. Based on this, the objective function can be formulated as:

$$\max_{\mathbf{P}, \mathbf{W}} \quad R_{\text{sum}} \tag{7a}$$

$$\text{s.t.} \quad C_1 : R_{\text{sum}} \geq R_{\text{min}}, , \tag{7b}$$

$$C_2 : ||w_k||^2 \leq 1, \forall k \in \{1, \ldots, N_K\}. \tag{7c}$$

IV. Numerical Results

Figure 2 shows the results for VQC in different circuits. The results indicate that VQC Circuit 1 has a lower training loss and achieves better convergence compared to VQC Circuit 2. The quantum operations were performed using IBM Qiskit. The simulation parameters were defined as follows: $\alpha = 0.01$, $N_{\text{shot}} = 1024$, $L_{\text{layer}} = 2$.

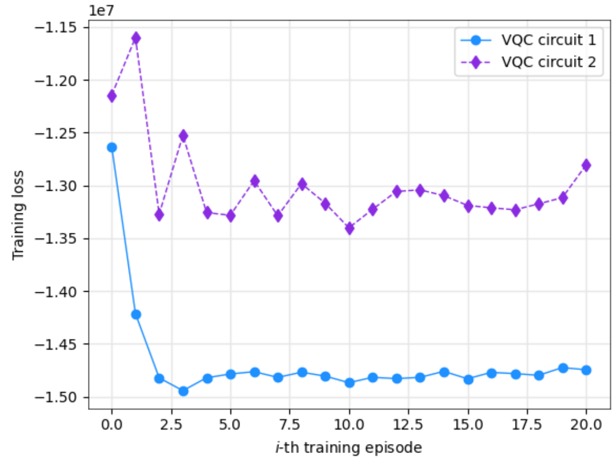


Figure 2: The training loss.

V. Conclusion

This study presents the performance analysis of VQC with different circuits. As a particular case, the VQC is applied to optimize the precoder-combiner in MIMO systems to maximize the achievable sum rate. The results show that VQC with Circuit 1 has a lower learning loss compared to VQC Circuit 2, resulting in a higher achievable sum rate.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education(No. 2022R1I1A1A01069334) This work was supported by Institute of Information & communications Technology Planning & Evaluation (IITP) grant-funded by the Korea government(MSIT) (No. 2021-0-02120, Research on Integration of Federated and Transfer learning between 6G base stations exploiting Quantum Neural Networks).

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