

# Performance Analysis of Active Reconfigurable Intelligent Surface-Assisted Cell-Free Massive MIMO with Hardware Impairment

Segbey Isaac\*, Prince Anokye†, Kyoung-Jae Lee\*

Hanbat National University, Daejeon, South Korea\*, Department of Mathematics and Electrical Engineering, IMT-Atlantique, Brest, France†.

segbeyi@edu.hanbat.ac.kr\*, princemcanokye@yahoo.com†, kyoungjae@hanbat.ac.kr\*

## Abstract

This paper analyzes the uplink (UL) spectral efficiency (SE) performance of an active reconfigurable intelligent surface (RIS)-assisted cell-free (CF) massive multiple-input multiple-output (mMIMO) under the influence of spatial correlation. The RIS reflecting elements (REs) are affected by phase noise. All access points (APs) and user equipment (UE) suffer from hardware impairments (HWIs).

## I . Introduction

CF mMIMO promises high SE, uniform quality of services, and does not require handover in cell areas. However, CF mMIMO cannot ensure a good quality of service under harsh channel conditions. RIS is capable of dynamically altering wireless channels by modifying the signal reflection using low-cost REs to optimize the communication performance [1]. It is thus envisaged that the new RIS-aided hybrid wireless network will accomplish sustainable capacity growth cost-effectively in the future. Due to double-fading, large sizes of passive RIS are utilized to enhance the wireless channel. This is, however, not sustainable due to limited space. Deploying a modest active RIS relatively improves the SE. Unfortunately, these technologies are imperfect which degrades the SE. This paper aims to analyze the achievable UL SE performance of an active RIS assisted CF mMIMO considering the impact of spatial correlation and phase noise at the RIS and HWIs at the APs and UEs.

## II . System Model

We consider an RIS aided CF mMIMO system, where  $M$  single-antenna APs are scattered in a wide area to jointly serve few  $K$  single-antenna users. To aid the communication between the APs and users, an RIS with  $N$  active REs is deployed. To decrease the implementation cost and power consumption, we assume both the APs and Users are equipped with low-quality hardware which results in HWIs. Also, the phases of the active RIS are affected by phase noise. We assume the standard block fading model. The following definitions are used:  $g_{mk}$  is the direct channel between the user  $k$  and the AP  $m$ ;  $\mathbf{h}_m \in \mathbb{C}^N$  is the channel between the AP  $m$  and the RIS; and  $\mathbf{u}_k \in \mathbb{C}^N$  is the channel between the RIS and the user  $k$ . Both  $\mathbf{h}_m$  and  $\mathbf{u}_k$  form a cascaded channel. In this work, we consider a more realistic channel model similar to [1]. We modeled  $g_{mk} \sim \mathcal{CN}(0, \beta_{mk})$ ,  $\mathbf{h}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_m)$  and  $\mathbf{u}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_k)$ , where  $\beta_{mk}$  is the large-scale fading coefficient;  $\mathbf{R}_m \in \mathbb{C}^{N \times N}$  and  $\mathbf{Q}_k \in \mathbb{C}^{N \times N}$  are the covariance matrices that characterize the spatial correlation among the channels of the RIS elements. We define  $\mathbf{R}_m = \alpha_m d_H d_V \mathbf{R}$  and  $\mathbf{Q}_k = \rho_k d_H d_V \mathbf{R}$ , where  $\alpha_m$  and  $\rho_k$  are the large-scale fading coefficients. Each element of the RIS

is  $d_H \times d_V$ , with  $d_H$  being the horizontal width and  $d_V$  being the vertical height of each RIS element.  $\mathbf{R} \in \mathbb{C}^{N \times N}$  and  $[\mathbf{R}]_{m'n'} = \text{sinc}(2 \|\mathbf{u}_{m'} - \mathbf{u}_{n'}\|/\lambda)$  where  $\lambda$  is the wavelength.  $\mathbf{u}_y, y \in \{m', n'\}$  is given by  $\mathbf{u}_y = [0, \text{mod}(y-1, N_H)d_H, [(y-1)/N_H]d_V]^T$  where  $N_H$  and  $N_V$  denote the total number of RIS elements in each row and column, respectively. Also, we define  $\Theta = \text{diag}([\sqrt{\xi_1}e^{j\theta_1}, \dots, \sqrt{\xi_N}e^{j\theta_N}]^T) \in \mathbb{C}^{N \times N}$  as the diagonal matrix containing the reflection coefficients of the active RIS, where  $\xi_n$  and  $\theta_n \in [-\pi, \pi]$  are the amplitude and phase of the  $n$ -th REs, respectively. The phase error is  $\bar{\Theta} = \text{diag}([\sqrt{\xi_1}e^{\bar{\theta}_1}, \dots, \sqrt{\xi_N}e^{\bar{\theta}_N}]^T) \in \mathbb{C}^{N \times N}$ , which is modeled according to the Von Mises distribution with zero mean and a characteristic function as  $\mathbb{E}\{\sqrt{\xi_n}e^{\bar{\theta}_n}\} = \frac{I_1(\kappa_{\bar{\theta}})}{I_0(\kappa_{\bar{\theta}})}$ , where  $I_j(\kappa_{\bar{\theta}})$  is the modified Bessel function of the first kind and order  $j$ . The aggregated channel between the  $m$ -th AP and the  $k$ -th user with  $\tilde{\Theta} = \Theta \bar{\Theta}$  is modeled as

$$v_{mk} = g_{mk} + \mathbf{h}_m^H \tilde{\Theta} \mathbf{u}_k \quad (1)$$

## III. Spectral Efficiency Analysis

During the data transmission phase, all the users are assumed to transmit their signals simultaneously to the APs via the active RIS and direct links. The received signal can be expressed as

$$y_m = \sqrt{\kappa_m} \left\{ \sum_{k=1}^K v_{mk} (\sqrt{p_k \kappa_k} s_k + \omega_k) \right\} + \omega_m + \mathbf{h}_m^H \tilde{\Theta} \mathbf{n}_A + n_m \quad (2)$$

where  $p_k, s_k \sim \mathcal{CN}(0, 1)$ ,  $n_m \sim \mathcal{CN}(0, \sigma_m^2)$  and  $\mathbf{n}_A \sim \mathcal{CN}(\mathbf{0}, \sigma_A^2 \mathbf{I}_N)$  denote the transmit power of  $k$ -th user, data of the  $k$ -th user, noise at the  $m$ -th AP, and noise at the active RIS, respectively.  $\kappa_k \in [0, 1]$  and  $\kappa_m \in [0, 1]$  are the hardware quality factors associated with  $m$ -th AP and  $k$ -th UE, respectively. The parameters  $\omega_k$  and  $\omega_m$  represent the HWIs that arise from the transmitting  $k$ -th UE to the receiving  $m$ -th AP. According to [2], we have  $\omega_k \sim \mathcal{CN}(0, p_k(1 - \kappa_k))$  and  $\omega_m | \{v_{mk}\} \sim \mathcal{CN}(0, (1 - \kappa_m) \sum_{k=1}^K p_k |v_{mk}|^2)$ .

The  $m$ -th AP applies the receive filter  $v_{mk}^*$  before forwarding the received signal to the CPU for further processing. We consider maximum ratio combining (MRC) at the APs. Thus, the signal obtained at the CPU for the  $k$ -th user is written as

$$r_k = \sum_{m=1}^M v_{mk}^* y_m \quad (3)$$

In this paper, we assume perfect channel state information. To obtain the SE expressions, we use the use-and-then-forget (UaTF) method [3]. The received signal for the  $k$ -th user is rewritten as

$$\begin{aligned} r_k = & \sqrt{p_k \kappa_k} \mathbb{E} \left\{ \sum_{m=1}^M \text{desired signal} \sqrt{\kappa_m} v_{mk}^* v_{mk} \right\} s_k \\ & + \sqrt{p_k \kappa_k} \left( \sum_{m=1}^M \sqrt{\kappa_m} v_{mk}^* v_{mk} - \mathbb{E} \left\{ \sum_{m=1}^M \text{BUG} \sqrt{\kappa_m} v_{mk}^* v_{mk} \right\} \right) s_k \\ & + \sum_{k' \neq k}^K \sum_{m=1}^M \sqrt{p_{k'} \kappa_{k'} \kappa_m} v_{mk}^* v_{mk'} s_{k'} \\ & + \sum_{k'=1}^K \sum_{m=1}^M \sqrt{\kappa_{k'} \omega_{k'}} + \text{receive impairment} \\ & + \sum_{m=1}^M \sum_{k'=1}^K v_{mk}^* \bar{n}_{A,m} + \sum_{m=1}^M v_{mk}^* n_m, \quad (4) \end{aligned}$$

Where  $\bar{n}_{A,m} = \mathbf{h}_m^H \bar{\Theta} \mathbf{n}_A$ ; the terms in (4) denote the desired signal, beamforming uncertainty gain (BUG), multi-user interference (MUI), transmit impairment, receive impairment, RIS amplified noise and amplified thermal noise at the APs, respectively. Using (4), the UL signal-to-interference-and-noise-ratio (SINR) for the  $k$ -th user is given by

$$\gamma_k = \frac{\kappa_k p_k \mathbb{E} \left\{ \sum_{m=1}^M \sqrt{\kappa_m} v_{mk}^* v_{mk} \right\}^2}{\kappa_k p_k B_k + \sum_{k' \neq k}^K \kappa_{k'} p_{k'} I_{k'} + Z_{A,k} + Z_{m,k} + \sum_{k'=1}^K T_{k'} + H_{m,k}} \quad (5)$$

where  $B_k, Z_{m,k}, I_{k'}, Z_{A,k}, T_{k'}$  and  $H_{m,k}$  denote the powers of the BUG, noise at APs, MUI from the  $k'$ -th user to the  $k$ -th user, amplified active RIS noise, transmit distortion term and receive distortion term. They are defined as

$$\begin{aligned} B_k & \triangleq \mathbb{E} \left\{ \left| \sum_{m=1}^M \sqrt{\kappa_m} v_{mk}^* v_{mk} - \mathbb{E} \left\{ \sum_{m=1}^M \sqrt{\kappa_m} v_{mk}^* v_{mk} \right\} \right|^2 \right\}, \\ Z_{m,k} & \triangleq \sum_{m=1}^M \sigma_m^2 \mathbb{E} \{ |v_{mk}|^2 \}, \\ I_{k'} & \triangleq \mathbb{E} \left\{ \left| \sum_{m=1}^M \sqrt{\kappa_m} v_{mk}^* v_{mk'} \right|^2 \right\}, Z_{A,k} \triangleq \sigma_A^2 \sum_{m=1}^M \mathbb{E} \{ \| v_{mk}^* \mathbf{g}_m^H \bar{\Theta} \|^2 \}, \\ T_{k'} & \triangleq \mathbb{E} \left\{ \left| \sum_{m=1}^M \sqrt{\kappa_m} v_{mk}^* v_{mk} \omega_{k'} \right|^2 \right\}, H_{m,k} \triangleq \mathbb{E} \{ | \sum_{m=1}^M v_{mk}^* \omega_m |^2 \} \end{aligned}$$

With (5) the SE is given by  $\text{SE}_k = 0.5 \log_2(1 + \gamma_k)$ .

#### IV. Simulation Results

The simulation results are obtained using the following values:  $p_k = 10 \text{ dBm } \forall k$ ,  $\sigma_m^2 = -92 \text{ dBm}$ ,  $\sigma_A^2 = -114 \text{ dBm}$ ,  $\xi_n = 10 \text{ dB}, \forall n$ ,  $d_h = d_v = \lambda/4$ ,  $\kappa_{\bar{\theta}} = 2$

We illustrate the impact of increasing the number of APs  $M$  on the SE and varying degrees of HWIs. as seen in Fig. 1. The UL achievable sum SE grows monotonically as  $M$  increases due to more multiplexing and array gains. The ideal hardware outperforms both non-ideal hardware and CF mMIMO without RIS. Using non-ideal hardware i.e.  $\kappa_k = \kappa_m = 0.97$  leads to the degradation in SE which still outperforms CF mMIMO without RIS system.

Fig. 2. shows a plot of the sum SE versus varying number of elements of the RIS for different degrees of HWI. The ideal hardware i.e.  $\kappa_k = \kappa_m = 1$  outperforms the non-ideal hardware which can be inferred from (5). The CF mMIMO without RIS system is unaffected by the number of RIS REs. The sum SE rapidly grows with the increase of  $N$  for smaller  $N$  but begins to saturate as  $N$  becomes larger. When  $N$  is lower, increasing  $N$

provides more paths to improve the cascaded channel, as  $N$  increases there are more interfering signals.

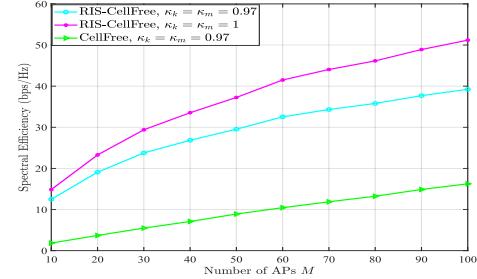


Fig 1: UL sum SE vs Number of APs with varying degrees of HWI ( $K = 5, N = 400$  ).

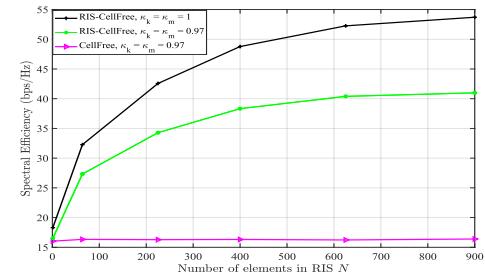


Fig 2: UL sum SE vs Number of active RIS REs under various degrees of HWIs ( $M = 100$  and  $K = 5$ ).

#### V. Conclusion

It is shown that active RIS significantly enhances the UL SE of active RIS-assisted CF mMIMO system with HWI at the APs, UEs and phase noise at the RIS despite the added noise.

#### ACKNOWLEDGMENT

This work was supported in part by the Institute of Information & Communications Technology Planning & Evaluation (IITP) grant through Korea Government (MSIT) under Grant 2021-0-00841 (Rate-splitting multiple access based cell-free extremely massive MIMO full-duplex transmission systems), and the ICAN (ICT Challenge and Advanced Network of HRD) program (IITP-2024-RS-2022-00156212).

#### REFERENCES

- [1] T. Van Chien, H. Q. Ngo, S. Chatzinotas, M. Di Renzo, and B. Ottersten, "Reconfigurable intelligent surface-assisted cell-free massive MIMO systems over spatially-correlated channels," *IEEE Transactions on Wireless Communications*, vol. 21, no. 7, pp. 5106–5128, 2022.
- [2] Y. Zhang, H. Zhao, W. Xia, W. Xu, C. Tang, and H. Zhu, "How much does reconfigurable intelligent surface improve cell-free massive MIMO uplink with hardware impairments?" *IEEE Transactions on Communications*, vol. 71, no. 11, pp. 6677–6694, 2023.
- [3] P. Anokye, S. Shin, W. Theodore, D. K.-P. Asiedu, and K.-J. Lee, "Active reconfigurable intelligent surface-assisted cell-free massive MIMO," in *2023 VTS Asia Pacific Wireless Communications Symposium (APWCS)*, 2023, pp. 1–5.