Outage Probability Analysis in RSMA Power Line Communications

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Abstract

This paper studies the outage probability (OP) performance of power line communication (PLC) system with rate splitting multiple access (RSMA). We consider a PLC system equipped with single-antenna source node that transmits information to two user nodes. We specifically derive analytical solutions to the OP of the common and private streams of the users and show the impact of power allocation to the OP of the common and private streams.

Ⅰ. Introduction

Power line communication (PLC) involves deploying existing power lines to support low and high data rate applications. The PLC system reduces the cost of implementation since it is deployed over existing power lines. PLCs operate in a broadcast manner, making it essential to improve the system's reliability [1]. Rate splitting (RS) multiple access (RSMA) on the other hand is a promising technology for next generation communication that is robust against interference. RS splits original messages intended for both users into common and private parts, encodes the common messages into a single stream and private messages independently [2]. At the receiver, successive interference cancellation (SIC) is applied to cancel the decoded common stream before the user decodes the private streams. This paper studies the outage probability (OP) of the common and private streams in a PLC system to examine the reliability of a RS supported PLC system.

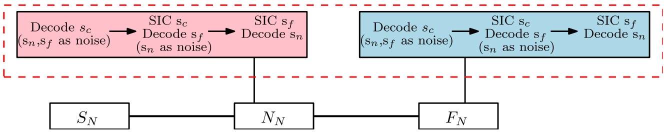

Fig. 1 System model of RSMA PLC network.

Ⅱ. System model

The PLC network consist of a source node S_N , two user nodes, near (N_N) and far (F_N) user nodes. The source node broadcasts a superimposed signal encompassing the common and private messages with different power allocations. The S-N and S-F links are denoted by the h_{sn} and h_{sf}, respectively. The PLC links are modeled as correlated log-normal random variables with distribution $h_m \sim \ln\mathcal{N}\left(\mu_{hm}, \sigma_{hm}^2\right)$ where $m \in (sn, sf)$ [1],[3]. The signal attenuation is modeled by $\alpha_m = \exp\left(-(b_o + b_1 f^k)\, d_m\right)$ where d_m is the link distance, f denotes the transmit frequency in MHz, k, b_o and b_1 are the attenuation constants. The effect of background and impulsive noises are considered by utilizing the Bernoulli-Gaussian random process. The total noise at any receiving node n_r, $\forall_r$, $\in$, (n, f), is given as $n_r = n_B + n_P n_I$, where n_r, n_B , and n_I, denote the total, background and impulsive noise with variances σ_r^2 , σ_B^2, and σ_I^2, respectively. n_P occurs with a probability of p indicating the probability of occurrence of impulsive noise. S_N broadcasts a superimposed signal,

$$x = \sqrt{a_c P_S} s_c + \sqrt{a_{np} P_S} s_n + \sqrt{a_{fp} P_S} s_f, \qquad (3)$$

where a_c, a_{np}, and a_{fp}, represent the fraction of power allocated to the common, near and far user private streams, respectively, with $a_c + a_{np} + a_{fp} = 1$ and $a_p = a_{np} + a_{fp}$. The received signals at the near and far users are given by (2) and (3), respectively as

$$y_n = \left(\sqrt{a_c P_S} s_c + \sqrt{a_{np} P_S} s_n + \sqrt{a_{fp} P_S} s_f\right)\alpha_{sn} h_{sn} + n_n, \qquad (2)$$

$$y_f = \left(\sqrt{a_c P_S} s_c + \sqrt{a_{np} P_S} s_n + \sqrt{a_{fp} P_S} s_f\right)\alpha_{sf} h_{sf} + n_f, \qquad (3)$$

where n_n and n_f denote the noises at the near and far users, with variances σ_n^2 and σ_f^2, respectively.

The SNRs for decoding s_c , s_f and s_n at the near user are, respectively, written as

$$\gamma_{c,nj} = \frac{a_c P_s \alpha_{sn}^2 h_{sn}^2}{a_p P_s \alpha_{sn}^2 h_{sn}^2 + \sigma_{nj}^2}, \gamma_{pn\to f,j} = \frac{a_{fp} P_s \alpha_{sn}^2 h_{sn}^2}{a_{np} P_s \alpha_{sn}^2 h_{sn}^2 + \sigma_{nj}^2},$$

$$\gamma_{p,nj} = \frac{a_{np} P_s \alpha_{sn}^2 h_{sn}^2}{\sigma_{nj}^2}.$$

The SNRs for decoding s_c and s_f at the far user are, respectively, written as

$$\gamma_{c,fj} = \frac{a_c P_s \alpha_{sf}^2 h_{sf}^2}{a_p P_s \alpha_{sf}^2 h_{sf}^2 + \sigma_{fj}^2}, \gamma_{p,fj} = \frac{a_{fp} P_s \alpha_{sf}^2 h_{sf}^2}{a_{np} P_s \alpha_{sf}^2 h_{sf}^2 + \sigma_{fj}^2}.$$

Ⅲ. Outage Probability Analysis

In this analysis, $\theta_c = 2^{Rc}$, $\theta_n = 2^{Rn}$, $\theta_f = 2^{Rf}$ $a_1 = P_S \alpha_{sn}^2$, $a_2 = P_S \alpha_{sf}^2$, $X = h_{sn}^2$, $Y = h_{sf}^2$. The OP of the common stream is written as

$$P_c^{out} = \sum_{j=1}^{2} p_j \; P_{cj}^{out}, \qquad (4)$$

where $P_{cj}^{out} = 1 - \Pr\left(R_{c,nj} > R_c\right) \Pr\left(R_{c,fj} > R_c\right)$. Also,

$$R_{c,nj} = \log_2\left(1 + \gamma_{c,nj}\right) = \log_2\left(1 + \frac{a_c P_S \alpha_{sn}^2 h_{sn}^2}{a_p P_S \alpha_{sn}^2 h_{sn}^2 + \sigma_{nj}^2}\right) \qquad (5).$$

$$Pr\left(R_{c,nj} > R_c\right) = Pr\left[\log 2\left(1 + \frac{a_c a_1 X}{a_p a_1 X + \sigma_{nj}^2}\right) > R_c\right],$$

$$= Pr\left[\frac{a_c a_1 X}{a_p a_1 X + \sigma_{nj}^2} > \theta_c - 1\right] = Pr\left[X > \frac{\sigma_{nj}^2(\theta_c - 1)}{a_1(1 - a_p\theta_c)}\right] \qquad (6).$$

$$R_{c,fj} = \log_2\left(1 + \gamma_{c,fj}\right) = \log_2\left(1 + \frac{a_c P \alpha_{sf}^2 h_{sf}^2}{a_p P \alpha_{sf}^2 h_{sf}^2 + \sigma_{fj}^2}\right) \qquad (7).$$

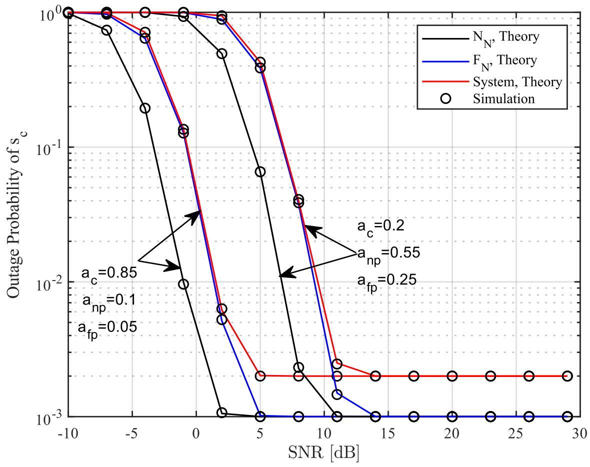

Fig. 2. OP of common streams versus SNR for different power allocations ($R_n = R_f = 0.1$ bps/Hz, p=0.001).

$$Pr\left(R_{c,fj} > R_c\right) = Pr\left[log\,2\left(1+\frac{a_c a_2 Y}{a_p a_2 Y+\sigma_{fj}^2}\right) > R_c\right],$$

$$= Pr\left[\frac{a_c a_2 Y}{a_p a_2 Y+\sigma_{fj}^2} > \theta_c - 1\right] = Pr\left[Y > \frac{\sigma_{fj}^2(\theta_c-1)}{a_2(1-a_p\theta_c)}\right] \qquad (8).$$

We stick to a similar approach to obtain the outage probability of the private streams as follows. The OP of the near user's private stream s_n is

$$P_n^{out(1)} = \sum_{j=1}^{2} p_j\, P_{nj}^{out}, \quad (9)$$

where $P_{nj}^{out} = 1 - Pr\left(R_{p,nj} > R_n\right)$. Also,

$$Pr\left(R_{p,nj} > R_n\right) = Pr\left[X > \frac{\sigma_{nj}^2(\theta_n-1)}{a_{np}a_1}\right]. \qquad (10)$$

The OP of the far user's private stream s_f is

$$P_f^{out(1)} = \sum_{j=1}^{2} p_j\, P_{fj}^{out}, \quad (11)$$

where $P_{fj}^{out} = 1 - Pr\left(R_{p,fj} > R_f\right)Pr\left(R_{pn\rightarrow fj} > R_f\right)$. Also,

$$Pr\left(R_{p,fj} > R_f\right) = Pr\left[Y > \frac{\sigma_{fj}^2(\theta_f-1)}{a_2(a_p-a_{np}\theta_f)}\right], \qquad (12)$$

$$Pr\left(R_{pn\rightarrow fj} > R_f\right) = Pr\left[X > \frac{\sigma_{nj}^2(\theta_f-1)}{a_1(a_p-a_{np}\theta_f)}\right]. \qquad (13)$$

The analytical solutions to all the probabilities are computed based on log-normal distribution. Details are however omitted due to space limitations.

IV. Simulation Results

Results are obtained using the parameters: $b_0 = 9.4x10^{-3}$, $b_1 = 4.2x10^{-7}$, $d_{sn} = 70$ m, $d_{sf} = 100$ m, $f = 30\,MHz$, $\sigma_I^2 = -15$dB, $a_c = 0.15, a_{np} = 0.2, a_{fp} = 0.65, \mu h_m = 3\,dB$, and $\sigma_{hm}^2 = 2\,dB$. The transmit power is given by $P_s = -50 + 10\,log_{10}(f)\;dBm$.

Fig. 2 shows the OP versus the SNR for the common stream of the system and OP of the common stream at the user nodes separately. It is observed that the theoretical results are tight relative to the simulation results. Generally, the OP decreases with increasing SNR. The OP of s_c at N_N reveals better performance compared to that at F_N.The OP performance improves when a larger proportion of the power is allocated to the common streams even in the low SNR region. The outage of the common streams can be reduced by increasing the a_c and/or the SNR. The common OP is limited by OP of s_c at F_N in the low to medium SNR region.

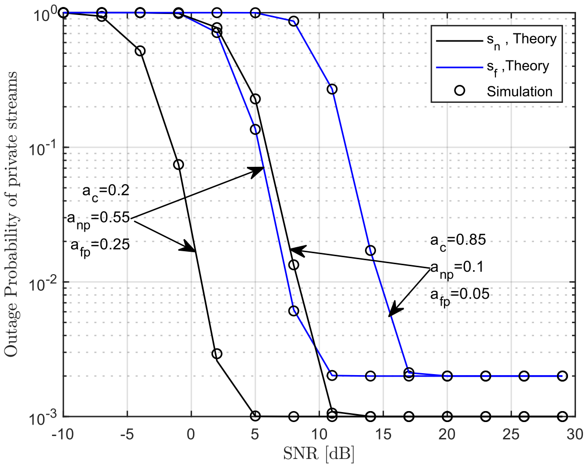

Fig. 3 OP of private streams versus SNR for different power allocations ($R_n = R_f = 0.1$ bps/Hz, p=0.001).

Fig. 3 compares OP over SNR for different power allocation scenarios. It is seen that the theoretical results are consistent with the simulation results. The OP is very high initially at low SNR but begins to fall from medium SNR as the SNR rises. The possibility that outage of s_n occurs is lower than s_f. This is possible due to high power allocation to the near user node and relative lower channel attenuation at N_N due to shorter d_{sn}. It is observed that the outage of the private streams can be significantly reduced by increasing the SNR and/or the power allocation to the private streams.

V. Conclusion

This paper has analyzed the OP of the PLC system with RSMA to explore the reliability of transmission of the common and private streams. The accuracy of our analysis is shown by comparing theoretical and simulation results which is shown to be exact. The OP of the common stream improves by allocating higher power to the common stream. Similarly, assigning high a_{np} enhances outage performance of the private streams.

ACKNOWLEDGMENT

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