

# Performance Measurement of Cooperative Terahertz Multicell Networks with Imperfect Beam Alignment: A Curve-Fitting Approach

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**Abstract**—Lately, in terahertz (THz) cellular networks, stochastic geometry framework has been employed for deriving coverage performance. Even if the stochastic geometry model is good for the purpose of mathematical tractability, it suffers from being unrealistic because the analytical model is based on many simplified assumptions i.e., the assumption is mainly limited to Poisson Point Process (PPP) and simple propagation channels which are often not applicable. And deriving analytical expression for a more realistic scenario may be quite difficult. Hence, in this paper we take on a simple yet more realistic model via a polynomial function and a parameterized logistic polynomial function to approximate the coverage in THz wireless networks with Non-Coherent Joint Transmission (NC-JT). Our study demonstrates that curve fitting using logistic function well approximates the performance of THz multicell networks.

**Index Terms**—Terahertz networks, beam misalignment, logistic function, regression.

## I. INTRODUCTION

The 6<sup>th</sup>-generation wireless systems are envisioned to go beyond 100 GHz into the THz band to increase data throughput enormously. With such high frequency, communication at THz band is largely sensitive to common blockages and it suffers from harsh path loss due to dispersion and atmospheric absorption. Thus, highly dimensional antenna arrays and dense base station deployments are required. Because of the resulting narrow beams, THz networks come at the cost of alignment errors. Therefore, there exists a tradeoff between beam gain and beam alignment. To reduce the impact of beam misalignment, macro diversity via BS cooperation is deployed in [1] where joint transmission from a few BSs enhances the coverage by mitigating intercell interference. For mathematical tractability, stochastic geometry was employed to model this network performance. However, performance analysis based on stochastic geometry is rather complex and is useful for limited scenarios. Hence, a simple yet applicable model is needed. Approximating the coverage probability of wireless network with the help of a parameterized logistic function has been done in [2] which can be applied to extensive scenarios and enables us to parameterize the performances. Following this method, this study demonstrates that a similar approach for performance approximation can be done in THz network as well.

## II. SYSTEM MODEL

Following the network model of [1], we consider the same wireless network over  $\mathbb{R}^2$  where BSs are distributed according to homogeneous PPP,  $\Phi_B$  with density,  $\lambda_B$ . The coverage probability is derived with respect to the typical UE located at the origin. For a BS at  $x \in \mathbb{R}^2$  w.r.t the origin, the probability that this BS is LoS is  $e^{-\beta\|x\|_2}$  and the pathloss gain is given by  $g_{L/N}(r) = Kr^{-\alpha}e^{-\gamma_{L/N}r}$ , which accounts for the decay of signal

power according to a distance and molecular absorption, where L and N denote LoS and NLoS, respectively. The small-scale fading between the  $i^{\text{th}}$  BS and the typical UE, i.e.,  $h_i$ , is modeled as independent Rayleigh fading. It is considered that  $M$  BSs with the strongest path loss gain w.r.t the typical user act as the serving BSs and thus form the cooperative cluster  $C_0$ . BSs antenna pattern is approximated with a sectored antenna model: within the 3dB beamwidth of  $\theta_b$ , the gain is  $G_b$ ; outside this sector, the gain is reduced by a factor of  $\zeta_b$ . The same pattern holds for the UE. A physically realizable gain pattern that accounts for  $M$  distinct beams in UE is used and the same expression holds for BSs by setting  $M=1$ :

$$G_u = \frac{2}{M(1 - \cos(\frac{\theta_u}{2})) + \xi_u(M \cos(\frac{\theta_u}{2}) - (M-2))}. \quad (1)$$

The alignment probability for BS and UE can be modelled using the Q-function as follows:

$$p_{b/u} = \mathbb{P}\left(|\Delta_{\theta_{b/u}}| \leq \frac{\theta_{b/u}}{2}\right) = \frac{1-2Q\left(\theta_{b/u}/\sqrt{2\sigma_{b/u}^2}\right)}{1-2Q\left(\pi/\sqrt{\sigma_{b/u}^2}\right)}. \quad (2)$$

The probability of interferer's main lobe being aligned with the UE is  $p_{ib} = \theta_b/2\pi$ , and that of UE's main lobe being aligned with an interferer is  $p_{iu} = M\theta_u/2\pi$ . The overall antenna gain for the  $i^{\text{th}}$  cooperating BS is a random variable  $G_u G_b Z_i^C$ . Likewise, the gain for the  $j^{\text{th}}$  interfering BS is  $G_u G_b Z_j^I$ . The  $Z^{C/I} \in \{1, \xi_u, \zeta_b, \xi_u \zeta_b\}$ , depends on the alignment probabilities from (2) and it accounts for the alignment error. All BSs are assumed to transmit the same power. The SINR experienced by the typical user is a random variable because the BS locations, channel gain and beam alignment errors are randomly sampled. The SINR of the cooperating BSs may be expressed as:

$$\text{SINR} = \frac{\sum_{R_i \in C_0} h_i \sqrt{Z_i^C g_{S(i)}(R_i)}^2}{I_L + I_N + \gamma}, \quad (3)$$

where  $C_0 = C_L \cup C_N$  denotes the set of cooperating LoS and NLoS BSs respectively,  $I_s = \sum_{R_j \in \Phi \setminus C_0} |h_j|^2 Z_j^I g_{S(j)}(R_j)$ , ( $\Phi = \Phi_L \cup \Phi_N$ ), and  $\Phi \setminus C_0$  = set of non-cooperative interfering BSs and  $s(i) \in \{L, N\}$  accordingly as the  $i^{\text{th}}$  link is LoS or NLoS.

During simulation, only a single BS cooperative cluster is considered which serves the typical UE and the rest of the BSs act as independent interferers. As per Lemma 1 of [1] this is the worst-case. So far, coverage probabilities with respect to threshold  $\tau$  has been estimated. Then, curve fitting using a non-linear least square method is performed to approximate the coverage probability of the typical UE with the given threshold  $\tau$ . In [2], simulation results have shown that curve-fitting results in more compact SINR models. Following the proposal in [2], the coverage probability using the parameterized logistic polynomial function is:

$$P(\tau) \approx \frac{1}{1 + \exp(-\beta_n \tau^n - \beta_{n-1} \tau^{n-1} - \dots - \beta_1 \tau - \beta_0)}, \quad (4)$$

where the values of  $\beta_n, \beta_{n-1}, \dots, \beta_0$  are obtained from curve fitting during Monte Carlo simulation. This result shall be used to approximate the probability of coverage for a typical user in our Terahertz network. Similar method has been implemented in [3] and [4]. The network is assumed to operate at frequency of 300 GHz. Using Monte Carlo simulation, the SINR performance of the proposed Terahertz network is obtained.

TABLE I  
SYSTEM PARAMETERS WITH VALUES

Notations	Parameters	Numerical Values
$\alpha_L, \alpha_N$	Pathloss Exponents	2, 2
$\gamma_L, \gamma_N$	Absorption Coefficients	$5e^{-3}m^{-1}, 5e^{-1}m^{-1}$
$\nu_L, \nu_N$	Fading Variances	1, 0.95
$1/\beta$	Blockage density	141.4 meters
$\theta_b, \theta_u$	BS and UE beamwidths	$3.17^\circ, 25.3^\circ$
$\zeta_b, \zeta_u$	BS and UE side lobe suppression factors	$1e^{-4}, 1e^{-3}$
$G_b, G_u$	BS and UE main lobe gains	35.35 dB, 18.79 dB
$P$	Transmit Power	10 dB
$\lambda_B$	Density of BS process	$1/(400\pi)$
$K$	Path loss intercept	$(c/4\pi f c)^2$ , $c=3e8ms^{-1}$
$\sigma_b, \sigma_u$	BS, UE alignment errors	$2^\circ, 10^\circ$

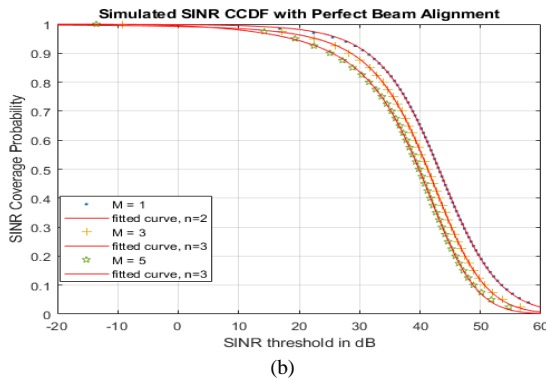
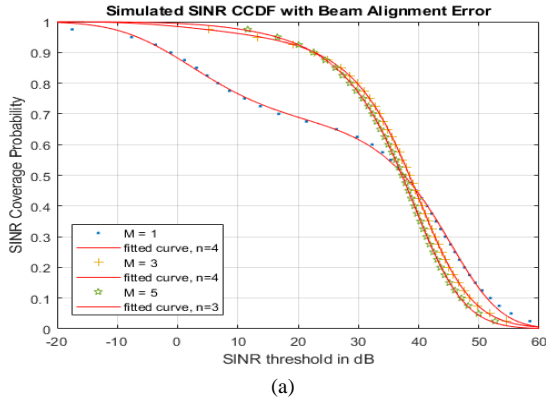


Fig. 1: (a) SINR coverage comparison between various values of  $M$  with beam alignment error along with their respective logistic polynomial fitted curves of  $n^{\text{th}}$  degree, (b) SINR coverage comparison for various  $M$  for perfect beam alignment case.

As can be observed in the figures, cooperative cluster from three BSs improves the coverage, and lesser returns are observed

for higher number of BSs because the increase in number of beams per UE increases its sensitivity to nearby interfering BSs. Increasing cooperative BSs also increases the probability for destructive interference within the cluster. Further, the beamforming gain per UE beam decreases with increase in  $M$ . During curve fitting, the maximum degree ( $n$ ) of the logistic function was  $n = 4$ . For  $M = 1, 2, 3, 4$ , and  $5$  in case of imperfect beam alignment,  $n$  took the values of  $4, 4, 4, 3$ , and  $3$ , respectively for the best possible fit. In case of perfect beam alignment for  $M = 1, 2, 3, 4$ , and  $5$ ,  $n$  took the values of  $2, 2, 3, 3, 3$ , respectively to get the best possible fit. On the other hand, we observed that fitting the curve using pure polynomial function requires very high degree, i.e., at least  $n=12$  to get a close fit for both perfect and imperfect beam alignment cases. Such high degree of the fitting function can easily lead to overfitting the data which is inefficient and increases fitting complexity.

### III. CONCLUSION

In this paper, we compared the coverages of BSs with and without beam alignment error and demonstrated that cooperation from a few BSs in THz network increases coverage. And we also described the process of approximating the coverage performance with a parameterized logistic polynomial function and found that it works efficiently for the THz networks with perfect and imperfect beam alignment. For future work, we would like to extend our curve fitting to deep neural network to determine the parameters  $\beta_n$  with the help of network parameter values. This would allow us to predict the coverage for any network without running Monte Carlo simulation.

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