사용자 연계 및 자원 할당을 위한 일반화된 고속 Water-Filling 알고리즘

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Generalized Water-Filling Algorithm for Fast User Association and Resource Allocation

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Abstract

We propose a generalized water-filling (WF) problem that contains weight coefficient, resource lower bound, and user association. Different from the conventional WF problem, the lower-bounded generalized WF problem requires user association for feasibility. However, the generalized WF problem becomes an intractable mixed-integer problem due to the binary user association variables. We solve the generalized WF problem by using Karush-Kuhn-Tucker condition and adopting an incremental user association scheme. The complexity of user association is $O(I^2)$, which is significantly lower than that of the exhaustive search, $O(2^I)$.

I. Introduction

Water-filling (WF) algorithm has been used to optimize resources for various communication systems, especially for maximizing mutual information in the OFDM system [1, 2]. However, few research works have considered a generalized WF algorithm with user association (or subcarrier allocation).

Motivated by optimization technique in [3], we generalize the WF algorithm by adding the minimum resource allocation constraint and weighted coefficient to the conventional water-filling problem.

II. Problem Formulation

We formulate the generalized WF problem as

$$\max_{\alpha_i, x_i, i\in\mathcal{I}} \sum_{i\in\mathcal{I}} a_i w_i \log(1+\lambda_i x_i)$$
$$\text{s.t.}\quad \textstyle\sum_{i\in\mathcal{I}} x_i \le P,$$
$$x_i \ge a_i r_i, i\in\mathcal{I}, \tag{1}$$

where $\mathcal{I}=\{1,\dots,I\}$; a_i is a binary user association variable; x_i is a variable for wireless resources; w_i is a weight coefficient; λ_i is a gain coefficient; r_i is a lower bound of the resource variable x_i; B is a resource capacity.

Binary user association parameter a_i has a value of 0 or 1. Variable a_i is adopted to make the problem feasible regardless of the lower bound r_i. We assume that $a_i w_i \log(1+\lambda_i x_i)$ is zero if $a_i = 0$.

One of the simple examples that show the necessity of user association variables is $a_i = 1$ and $r_i = P/2$ for all $i \in \mathcal{I} = \{1,2,3\}$. In this case, the minimum requirements already exceed the resource capacity. If we do not change the user association variables from 1 to 0, there is no feasible solution for the problem.

III. Proposed solution

We first find the optimal solution of the problem using the Karush-Kuhn-Tucker (KKT) condition, assuming that feasible $a_i, i\in\mathcal{I}$ are given. Then, we propose an incremental algorithm that can find the near-optimal combination of $a_i, i\in\mathcal{I}$.

A) Generalized Water-Filling Solution: When the feasible combination of $a_i, i\in\mathcal{I}$ is given, we can reformulate the proposed problem (1) as follows:

$$\max_{x_i, i\in\hat{\mathcal{I}}} \sum_{i\in\hat{\mathcal{I}}} w_i \log(1+\lambda_i x_i) \tag{2a}$$
$$\text{s.t.}\quad \textstyle\sum_{i\in\mathcal{I}} x_i \le P, \tag{2b}$$
$$x_i \ge r_i, i\in\hat{\mathcal{I}}, \tag{2c}$$

where set $\hat{\mathcal{I}}$ is defined as

$$\hat{\mathcal{I}} = \{i\in\mathcal{I} \mid a_i = 1\}. \tag{3}$$

The Lagrangian of Problem (2) is denoted as

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{\mu},\nu) = \textstyle\sum_{i\in\hat{\mathcal{I}}} w_i\log(1+\lambda_i x_i) + \sum_{i\in\hat{\mathcal{I}}}\mu_i(x_i - r_i) - \nu(\sum_{i\in\hat{\mathcal{I}}} x_i - P), \tag{4}$$

where vector $\boldsymbol{x}$ and $\boldsymbol{\mu}$ is defined as $\boldsymbol{x} = [x_1,\dots,x_I]$ and $\mu = [\mu_1,\dots,x_I]$.

At the global maximum, partial derivative of $\mathcal{L}(\cdot)$ must be zero. Then, we have

$$\nabla_{x_i}\mathcal{L}(\cdot) = \frac{w_i\lambda_i}{x_i} + \mu_i - \nu = 0. \tag{5}$$

From the KKT condition, variables satisfy the following equations at the global maximum:

$$\mu_i(x_i - r_i) = 0, \forall i\in\mathcal{I} \tag{6}$$

Combining (5) and (6) with respect to μ_i, we have

$$\left(\frac{w_i\lambda_i}{x_i} - \nu\right)(x_i - r_i) = 0, \forall i\in\mathcal{I}. \tag{7}$$

Because x_i should be larger than r_i, we can conclude

$$x_i = \max\left(r_i, \frac{\nu}{w_i\lambda_i}\right), \forall i\in\mathcal{I}. \tag{8}$$

It is noteworthy that the weight coefficient and gain have the same order in (8), so *they equally contribute to the allocation of the resources*.

Since equation (8) is monotonically increasing according to the variable ν, we can uniquely determine the optimal ν by the following equation:

$$\sum_{i\in\mathcal{J}} x_i = \sum_{i\in\mathcal{J}} \max(r_i, \frac{\nu}{w_i\lambda_i}) = P. \quad (9)$$

B) Fast User Association Algorithm: In this section, we propose a fast user association algorithm. We take the advantage of the property that the optimal solution of the generalized WF problem can be quickly computed using (8) and (9).

Our strategy is to check the result of adding one more user. Given an arbitrarily chosen and feasible user association set $\mathcal{J}_1$, we make a new set $\mathcal{J}_i$ that contains user i if the user does not belong to $\mathcal{J}_1$. Then, several sets $\mathcal{J}_i$ will be generated from $\mathcal{J}_1$.

We check whether set $\mathcal{J}_i$ violates (2b) as follows:

$$\sum_{i^*\in\mathcal{J}_i} r_{i^*} \leq P. \quad (10)$$

Let $i_1, \ldots, i_n$ be the index of newly generated feasible sets. Among set $\mathcal{J}_1$ and $\mathcal{J}_{i_1}, \ldots, \mathcal{J}_{i_n}$, we can determine which set gives the largest target function value (2a).

If we start from the empty set $\mathcal{J}_1 = \phi$, we can find the near-optimal user association set within an affordable time budget. Also, the above procedure is terminated when $\mathcal{J}_1$ does not change at all.

Compared with the exhaustive user association algorithm, the computational complexity significantly decreases from $O(2^I)$ to $O(I^2)$.

IV. Numerical Experiments

We apply the proposed water-filling algorithm to the bandwidth allocation scheme for the proportional fairness maximization problem. The system model and parameter configuration follow the problem proposed in [4].

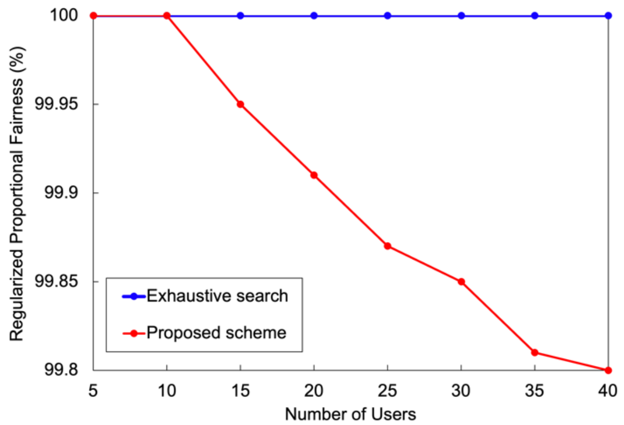

Fig. 1. Regularized proportional fairness. The y-axis value is regularized by the PF of the exhaustive search method.

Fig. 1 illustrates the regularized proportional fairness of the proposed method and exhaustive search. When the number of users is less than 10, the proposed scheme achieves the global optimal. Then, the proportional fairness gradually degrades 0.2 percent point (%p) when the number of users increases up to 40. When the number of users is more than 40, we fail to compute the proportional fairness of the exhaustive search scheme, due to the exponential increase in computational complexity.

V. Conclusion

We suggest a generalized WF problem with user association. The optimal solution of the proposed problem is obtained by using the KKT condition when the feasible user association set is given. Also, the proposed user association algorithm shows a negligible performance gap against the exhaustive search method, while significantly decreasing the computational complexity from $O(2^I)$ to $O(I^2)$.

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