Deep Learning Assisted Quantum State Tomography

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***Abstract*—This paper revisits the application of deep learning techniques to perform quantum state tomography on the obtained experimental measurement data from the noise model of cloud quantum computers provided by IBM. We propose a quantum-aware optimizer that can be efficiently employed to train the neural networks on any noise model. We show that the trained network can filter the noise from measurement data and can be used to reconstruct the quantum states with high fidelity.**

I. Introduction

Quantum information theory[1] is an emerging field of this era as its disparate paradigm of computing promises novel algorithms and computational speedups. Quantum computing employs the laws of physics such as superposition and entanglement for its operation. Although it has the potential to revolutionize many fields of science, it's still in its infancy and is prone to errors. The results we obtain from this error-prone device are probabilistic and to validate the correctness of the measurement results obtained from experiments on quantum computers we perform quantum tomography (QT). QT is a post-measurement technique that is used to benchmark noisy-intermediate scale quantum (NISQ) devices with applications ranging from quantum communication, quantum sensing, quantum computing, and so on [2], [3]. It provides a complete description of the imperfections in the NISQ devices. In this paper we look into quantum state tomography (QST) which comprises of two steps i) Measurement of a given unknown quantum state on some predefined basis to get the classical data ii) performing post-processing techniques on this obtained data to reconstruct the unknown quantum state.

The most commonly employed techniques in literature for performing QST are maximum likelihood estimation (MLE), Bayesian mean estimation (BME), and linear inversion [4], [5]. However, both MLE and BME are resource-intensive and more sensitive to noise. Therefore, significant research has been directed toward the estimation of unknown quantum states in the presence of noise [6]. This paper proposes the quantum state estimation technique using a lightweight and robust deep learning model. We show that the trained deep learning model leads us to the actual pure quantum state even in the presence of noise in NISQ devices.

II. Method

Any single qubit quantum state can be experimentally reconstructed using the Stokes parameters as [1]

$$\rho = \frac{1}{2}\sum_{i=0}^{3} s_i \sigma_i, \tag{1}$$

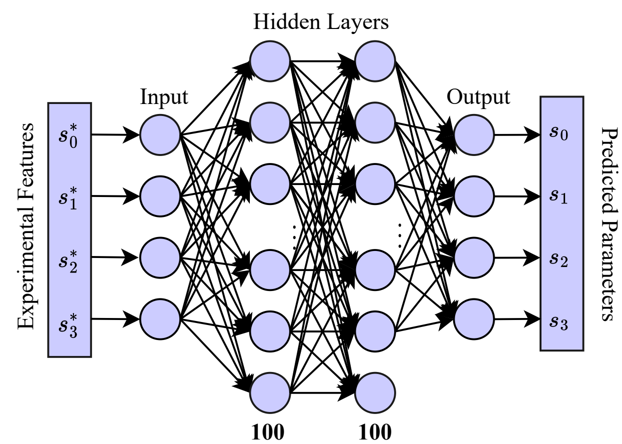

Figure 1. Deep Learning Model with two hidden layers each having 100 trainable weights for a single quantum state estimation. It takes the stokes parameters as input features and predicts the theoretical expectation values for quantum state reconstruction.

where s_i represents the Stokes parameters, σ_0, σ_1, σ_2, and σ_3 are the Identity, Pauli X, Pauli Y, and Pauli Z matrices, respectively. In general, we require d^2 parameters to reconstruct any n-qubit state, where $d = 2^n$.

A. Dataset Generation and Model Training

We first generate a dataset of 2000 randomly generated quantum states prepared on the noise model of a cloud-based quantum computer (IBMQ Manila) provided by IBM Quantum[7]. The dataset contains stokes parameters (inferred from measurement data on the real quantum computer) as input features and theoretical expectation values in Pauli bases as actual state output as shown in fig 2. We then add random Gaussian noise $\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ with $\mu = 0$ and $\sigma^2 = 0.2$ and split the dataset into training(70%), validation(20%), and testing data(10%). Then we train the deep learning model as shown in 1 on the noisy experimental training data to predict the theoretical results.

B. Model Configuration

1) Evaluation Metrics: we use mean squared error (MSE) as our loss function which is given by

$$J(W) = \sum_{i=1}^{D} (W^T \tilde{S}_i - S_i)^2 \tag{2}$$

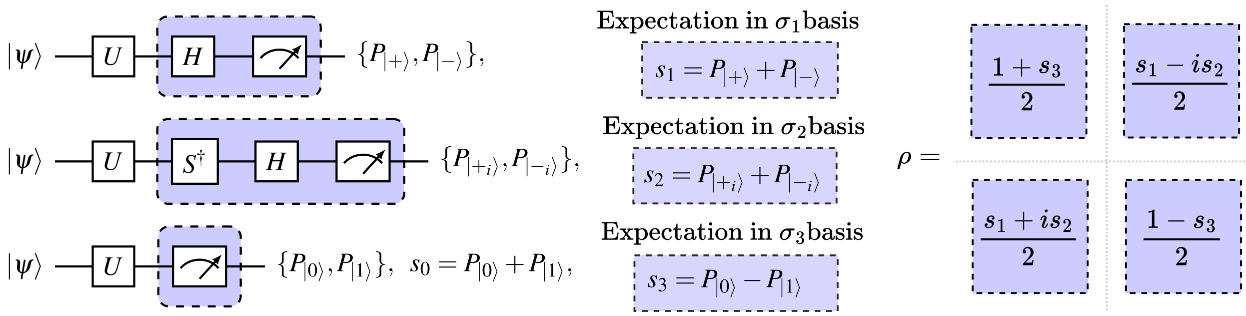

Figure 2. Quantum state reconstruction using stokes parameters.

where W is the weight matrix learned by the model, $\tilde{S}_i$ are the input stokes parameter vector, and S_i is the actual state expectation values vector. Moreover, we evaluate learning performance of the model by a metric known as Fidelity which is defined as;

$$F(\rho, \sigma) = \operatorname{tr}\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2. \tag{3}$$

2) Optimizer: We use the Stochastic Gradient Descent Optimisation Algorithm (SGD) and propose infidelity as its learning rate parameter to train the model. The modified SGD is given by,

$$w_{\text{new}} = w - \alpha \frac{\partial J(W)}{\partial w} \tag{4}$$

$$\alpha := 1 - F(\rho, \sigma) \tag{5}$$

where α is the learning rate which is updated by evaluating infidelity between the predicted state and actual state.

C. State Validation

We use eq. 1 to reconstruct the density matrix by inferring prediction parameters $\tilde{s}_i$ from our model. However, sometimes it has non-positive eigenvalues which makes it physically invalid. Hence, if we assume the state to be pure, we can make it valid by performing eigenvalue decomposition of the estimated density matrix and choosing the eigenvector as the final state corresponding to the highest eigenvalue[5].

III. Results

We obtain the mean fidelity of 200 test quantum states reconstructed using our trained model and compare it with the reconstruction results from the raw results of IBM Computer. As the data is subjected to random Gaussian noise, we perform the experiment 1000 times. Figure. 2 depicts that the deep learning model shows significant robustness in the presence of noise. On the other hand, standard reconstruction from IBM computers losses its accuracy when subjected to a moderate level of Gaussian noise strength.

IV. Conclusion

We have demonstrated the robust reconstruction of the pure quantum state using the noise model of IBM quantum computers. The given method can be used as a benchmark for the near-term quantum device, which usually suffers from depolarizing noise. Although we only show results for single qubit for simplicity this technique can also be applied to multi-qubit cases for any noise model of a quantum device as deep learning approaches are theoretically proven to universally approximate any function.

Method	min fidelity	max fidelity	mean fidelity
IBM	0.88	0.92	0.90
Proposed	0.96	0.99	0.98

Figure 3. Comparison of 200 reconstructed states' fidelity under Gaussian noise. Each state has been subjected to 1000 experiments having random noise. The first row depicts the fidelity of states reconstructed using raw measurement values, while in the second row, these values are first processed through the network, and then the states are reconstructed. It clearly shows that the network learns to filter the noise from obtained measurement values.

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