Variational Method for High-Fidelity Quantum State Preparation

Muhammad Mustafa Umar Gondel, Ahmad Farooq, Hyundong Shin
Department of Electronics and Information Convergence Engineering, Kyung Hee University, Korea
Email: hshin@khu.ac.kr

***Abstract*—Quantum state preparation is an important ingredient in successfully implementing higher-level quantum algorithms. The information is first encoded into a quantum state for a practical solution of real-world application to be run on the quantum computer. Noisy-intermediate scale quantum devices impose significant errors while preparing the quantum state. IBM has devised an optimization algorithm to design the quantum state for any given state vector. In this paper, we propose a variational method to prepare the quantum state with higher fidelity. Our result provides 10^4 times improvement over the existing method.**

I. INTRODUCTION

Quantum state preparation is fundamental to every application in quantum mechanics, whether it is computing, communication, or sensing [1], [2], [3]. Quantum information and computation theory has received increased consideration in the recent era due to the exponential speed-up over classical information processing in domains such as secure communication, quantum chemistry, and effectual implementation of certain computation tasks, such as prime number factorization [4]. Arbitrary quantum state preparation using unitary decomposition on available noisy-intermediate scale quantum (NISQ) devices limits the intended advantage of the quantum computer.

The imperfection of quantum gates causes noise in the state preparation process. IBM quantum computer uses superconducting materials to build a qubit. Microwave pulses control quantum gates. Qubit's interaction with the environment causes an error in the final state of the qubit when we apply a microwave pulse. We say this is the imperfections in the quantum gates which cause the noise at the output of any quantum gate. Quantum gates are noisy; hence inaccurate state yields. We can cater to the gate noise and involve feedback to improve the accuracy of the state being prepared [5], [6], [7]. The variational method is the core concept that we implemented in this paper. IBM uses its in-built methods to prepare required states using unitary decomposition. This in-built method is optimized for any unknown unit norm vector and provides an appropriate output quantum state.

In this paper, we show how the variational method can be used to find optimal parameters of universal single-qubit unitary that perform the desired transformation between input and output quantum states. We will also demonstrate that the quantum state prepared by our variational method has significantly lower infidelity than the state preparation obtained by the IBM built-in method.

II. METHOD

Any quantum state known as qubit can be represented by unit norm vector $|\psi\rangle$. A qubit can also be defined on the Bloch sphere. It is represented as

$$\rho = \frac{1}{2}(\sigma_o + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z) \tag{1}$$

where

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2}$$

and r_x, r_y, and r_z represent the real parameters. The norm of these real parameters should satisfy

$$r_x^2 + r_y^2 + r_z^2 \leq 1. \tag{3}$$

The state is pure if inequality in (3) is changed into equality; otherwise, the state is mixed.

Almost all modern NISQ architecture initialize the qubit state to be $|0\rangle$. To generate any particular single qubit quantum state $|\psi\rangle$, we require unitary operations

$$|\psi\rangle = U\,|\psi_o\rangle\,, \tag{4}$$

where $|\psi_o\rangle = |0\rangle$ and

$$U(\theta,\mu,\lambda) = \begin{bmatrix} cos(\frac{\theta}{2}) & -e^{i\lambda}sin(\frac{\theta}{2}) \\ e^{i\mu}sin(\frac{\theta}{2}) & e^{i(\mu+\lambda)}cos(\frac{\theta}{2}) \end{bmatrix}. \tag{5}$$

U matrix has three parameters θ, μ and λ. We are required to find these parameters to minimize the error between the target state and the output of the quantum circuit state. We use infidelity as our objective function and formulate our optimization problem as

$$\begin{aligned} \min_{|\phi\rangle} \quad & 1 - |\langle\psi|\phi\rangle|^2 \\ \text{subject to} \quad & \mathrm{tr}\,(|\phi\rangle\langle\phi|) = 1 \\ & |\phi\rangle\langle\phi| \geq 0, \end{aligned} \tag{6}$$

To calculate the objective function, we first apply parametrized random unitary $U(\theta,\mu,\lambda)$ on $|0\rangle$. We then perform quantum state tomography to reconstruct the desired quantum state [8], [9]. We use the standard state tomography algorithm to estimate the quantum state at the output of the quantum circuit. We measure in Pauli X, Y, and Z basis to

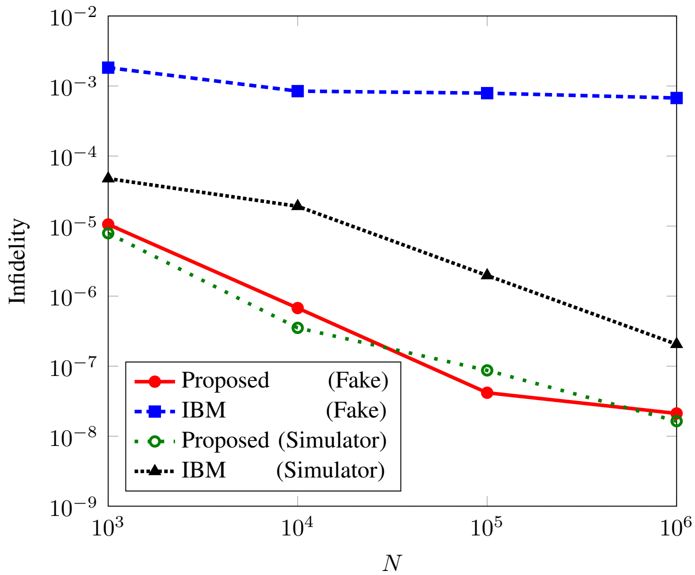

Figure 1. Infidelity plots of IBM and the proposed algorithm are plotted as a function of the number of shots. Both simulations are performed on the Aer simulator and fake Belam backend provided by IBM. We can see that the performance of our algorithm is significantly high.

estimate real parameters r_x, r_y and r_z. They are obtained from the probability distribution as follows

$$\begin{aligned} r_x &= \operatorname{tr}\left(\left|\phi\right\rangle\left\langle\phi\right| X\right) \\ &= \operatorname{tr}\left(\left|\phi\right\rangle\left\langle\phi\right|\left(\left|+\right\rangle\left\langle+\right| - \left|-\right\rangle\left|-\right\rangle\right)\right) \\ &= \left|\left\langle\phi|+\right\rangle\right|^2 - \left|\left\langle\phi|-\right\rangle\right|^2 \\ &= P_{|+\rangle} - P_{|-\rangle}, \end{aligned} \tag{7}$$

where $|+\rangle = \frac{|0\rangle+|1\rangle}{2}$ and $|-\rangle = \frac{|0\rangle-|1\rangle}{2}$. Similarly, we can obtain

$$\begin{aligned} r_y &= P_{|+_i\rangle} - P_{|-_i\rangle} \\ r_z &= P_{|1\rangle} - P_{|0\rangle}, \end{aligned} \tag{8}$$

where $|+_i\rangle = \frac{|0\rangle+i|1\rangle}{2}$ and $|-_i\rangle = \frac{|0\rangle-i|1\rangle}{2}$. After calculating these parameters, we use (1) to estimate the state. Since the output state of the IBM device is pure, we perform the spectral decomposition of the density matrix and select the state that corresponds to the highest eigenvalue $|\hat{\phi}\rangle$. We use the Powell as a classical optimizer and minimize the objective function of (6) to get the optimal parameter θ, μ, and λ.

III. Result

To compare the accuracy of our state preparation method, we use infidelity as the performance metric defined as

$$1 - F\left(\rho, \sigma\right) = 1 - tr\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2. \tag{9}$$

We generate 100 randomly pure quantum states from Haar measure. We theoretically calculate the quantum state, which is prepared by applying the single qubit unitary $U\left(\theta, \mu, \lambda\right)$ on $|0\rangle$ obtained from our method. We also obtained the quantum state by IBM-optimized built-in circuit decomposition of the unitary. We then plot the infidelity between the prepared state and target state obtained from our method and through the IBM optimization technique in Figure 1. We run an algorithm on fake backend Belam provided by IBM and Aer simulator. Both the fake backend and simulator show the mean value of infidelity for 100 states as a function of the number of shots N. Our algorithm significantly outperforms IBM's algorithm in terms of accuracy in preparing similar states on the fake backend and the simulator. Statistical comparison between IBM's algorithm and the proposed algorithm shows a ten-fold improvement in the accuracy of the state being prepared on Aer simulator backends. In contrast, on the Fake backend, it is far more for up to 10000 times improvement. In the fake backend, accuracy is high, but it is caused by a large value of N (shots), so it can not be performed on the real quantum computer in the near-future.

IV. Conclusion

We have proposed a fidel and more accurate technique for the quantum state preparation. The performance of the proposed scheme is significantly higher than the existing circuit-based quantum state preparation method. The proposed scheme can be used as a benchmark for quantum devices and pave the way toward achieving quantum supremacy over the classical system.

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