Reliable Implementation of Quantum Fourier Transform on NISQ Devices

Fadhel Hariz Dzulfikar, Muhammad Shohibul Ulum, Ahmad Farooq, and Hyundong Shin
Department of Electronics and Information Convergence Engineering, Kyung Hee University, Korea
Email: hshin@khu.ac.kr

***Abstract*—Shor algorithm is one of the most spectacular discoveries in quantum computing; that quantum computer can solve problems which is not feasible on a classical computer. Quantum Fourier Transform (QFT) is the key ingredient for factoring and many other interesting quantum algorithms. We demonstrate the implementation of QFT on the noise model of IBM superconducting quantum devices. We show that by utilizing the measurement error mitigation method, we can correct the solution of QFT for some use cases on the IBM quantum computing device.**

I. Introduction

Quantum computers can perform tasks exponentially faster than classical computers. Quantum computers represent a fundamentally new paradigm for processing information. A quantum bit, or a qubit, in short, is a quantum counterpart of the bit, which is a two-dimensional state vector that can be decomposed into a linear combination of orthonormal basis states—called quantum superposition. Shor's prime factorization algorithm is one of the most well-known algorithms that illustrate the capabilities of quantum computers. Solving this computational problem in a traditional computing environment would take billions of years, but in theory, a quantum computer might do it in a few hours. Another significant discovery is the quadratic speedup of the Grover quantum search algorithm over conventional algorithms for searching over an unsorted database.

Quantum Fourier Transform (QFT) is the important element used by both algorithms. The classical task of computing Fourier transforms of classical data is not sped up by applying the quantum Fourier transform on the quantum mechanical amplitude. However, one crucial task it does enable is phase estimation, which, under certain conditions, approximates the eigenvalues of a unitary operator and enables us to tackle other issues like factoring problems and quantum search algorithms [1]. However, operating devices for quantum processing have, and still do, face several challenges. We are in the age of noisy intermediate-scale quantum (NISQ) technology. Error mitigation has been proposed as a partial solution to this problem [2], [3].

In this paper, we examine a method for measurement error mitigation that utilizes the error probability matrix, and a linear equation system [4]. Then we implement this technique on an IBM quantum computer to demonstrate how error mitigation can reduce errors to find the correct solution of the 5-qubit use case on the real device.

II. Method

Quantum Fourier Transform (QFT) algorithm is an extension of the Discrete Fourier Transform (DFT) to the quantum domain, with exponential speed-up due to superposition and quantum parallelism. The QFT is fundamentally the same as the Fast Fourier Transform (FFT) in that it executes a DFT on a list of complex, but the output of the QFT is stored as amplitudes of a quantum state vector. The QFT is identical to DFT with a different notation and is defined by

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle \,, \tag{1}$$

where the amplitudes y_k are the DFT of the amplitudes x_j. With a little algebra, QFT can be defined in the binary representation

$$|j_1, \cdots, j_n\rangle \rightarrow \frac{\left(|0\rangle + e^{2\pi i 0.j_n} |1\rangle\right) \cdots \left(|0\rangle + e^{2\pi i 0.j_1 \cdots j_n} |1\rangle\right)}{2^{n/2}}. \tag{2}$$

We use a measurement error mitigation technique to remove the measurement error on the noise model of the quantum computing device. In this technique, we adjust the results on a measurement device using Positive-Operator Valued Measure (POVM) [5]. The noise model we defined as the POVM is identical to invertible classical post-processing of experimental data, and it can therefore be inverted merely by classical post-processing. Let's use the term M_{ideal} to designate an n-outcome quantum measurement that, in theory, ought to be connected to our measurement tool. In reality, some POVM M_{exp} serves as an actual measurement defining our equipment due to the existence of noise. In this model, we suppose that the relationship between M_{ideal} and M_{exp} is represented by

$$\forall i \quad M_i^{exp} = \sum_j p(i|j) M_j^{ideal}, \tag{3}$$

The left-stochastic property of Λ means that $\sum_i \Lambda_{i,j} = \sum_i p(i|j) = 1$. Let's refer to the ideal probability vector that would have been acquired in the perfect device as p_{ideal}, and the probability p_{exp} vector obtained in an experiment using a noisy device is given by

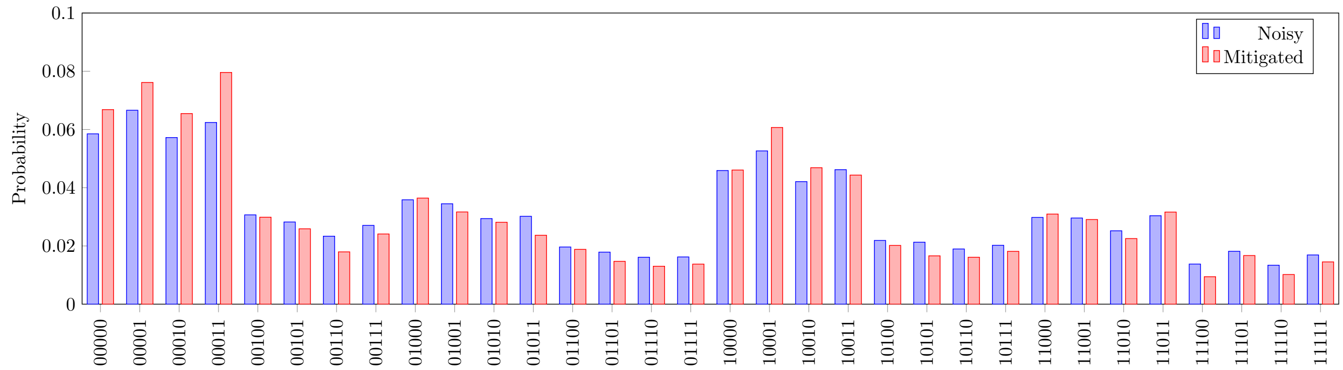

Fig. 1. Probability distribution as a function of binary integer of 5 qubits. We can see that error-mitigated methods correct the error on ibm_device and give us the desired result after implementing the Quantum Fourier Transform.

$$p_{exp} = \Lambda p_{ideal}, \tag{4}$$

According to this, one may left-multiply p_{exp} by its inverse to get the statistics one would have obtained from experiments on the ideal quantum computer.

$$p_{ideal} = \Lambda^{-1} p_{exp} \tag{5}$$

This allows us to mitigate the experimental noise through classical post-processing. The current IBM quantum devices has this measurement noise model

III. Results

We perform the QFT on IBM fake simulator device. This simulator makes use of actual quantum device noise data from IBM. We will specifically apply the ibmq_quito noise model in this experiment.

First, we created 5 qubits QFT circuit consisting of two types of gates. We use Hadamard and controlled rotation $CROT_k$ gate. They are defined as

$$\boldsymbol{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad CP(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & e^{i\theta} \end{bmatrix}, \tag{6}$$

where $\theta = \phi/2^{k-1}$.

Next, we encode a number in the qubits based on a computational basis. In this experiment, we use the number 3, defined by '00011' in binary. After that, we applied the QFT and inverse QFT to return the initial quantum state. To provide us with the ideal results that will serve as our benchmark for evaluating the effectiveness of the error mitigation method, this circuit is first implemented on the Qiskit Aer simulator. Afterward, we ran this circuit 10 times and averaged the result on the simulator with the ibmq_quito noise model. On each run of the circuit implemented on the simulator with and without the noise model, we used 1024 shots. We use measurement error mitigation for each run to compare the unmitigated and mitigated effects. From Fig. 1, in this case, we can see that the results from real hardware have errors and are different from what we see on the simulator without noise. On the other hand, the results gave the correct outcomes after applying measurement error mitigation.

IV. Conclusion

In this paper, we investigated a measurement error mitigation technique to reduce error when running a quantum circuit in a NISQ device. Additionally, we used the QFT algorithm as an example to show how to mitigate errors on an IBM quantum computer and demonstrated how, in some circumstances, classical post-processing could help reduce errors in NISQ-era devices.

Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2019R1A2C2007037 and 2022R1A4A3033401) and by the MSIT (Ministry of Science and ICT), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2021-0-02046) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation).

References

[1] M. A. Nielsen and I. L. Chuang, "Quantum computation and quantum information: 10th anniversary edition," *Cambridge University Press*, 2010.

[2] A. Farooq, U. Khalid, J. ur Rehman, and H. Shin, "Robust quantum state tomography method for quantum sensing," *Sensors*, vol. 22, no. 7, p. 2669, Mar. 2022.

[3] A. Farooq, M. A. Ullah, J. ur Rehman, K. Lee, and H. Shin, "Self-guided quantum state learning for mixed states," *Quantum Inf. Process*, vol. 21, no. 7, p. 243, Jul. 2022.

[4] F. B. Maciejewski, Z. Zimborás, and M. Oszmaniec, "Mitigation of readout noise in near-term quantum devices by classical post-processing based on detector tomography," *Quantum*, vol. 4, p. 257, Apr 2020.

[5] J. ur Rehman, A. Farooq, and H. Shin, "Discrete Weyl channels with Markovian memory," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 3, pp. 413–426, Mar. 2020.