

# Power Minimization of Intelligent Reflecting Surface-Aided Uplink Systems

Jiao Wu, Hyunsoo Kim, and Byonghyo Shim  
Seoul National University

{jiaowu, hskim, bshim}@islab.snu.ac.kr

## Abstract

Employing intelligent reflecting surface (IRS) is emerging as an alternative to massive antenna systems for improving signal quality and suppressing interference. Specifically, IRS is a planar surface consisting of a large number of low-cost reflecting elements each of which can reflect the incident signal independently with a desired phase shift, thus achieving the passive beamforming gain. In this paper, we study the uplink power control of an IRS-aided Internet of Things (IoT) network. Our goal is to minimize the uplink power by alternatively optimizing the IRS phase shifts and BS beamforming vectors, subject to the data requirement of users. To solve the formulated non-convex optimization problem, we develop an efficient scheme, called the Riemannian manifold-based alternating optimization (RM-AO).

## I. Introduction

The intelligent reflecting surface (IRS) has recently gained much attention since it can significantly enhance the spectral and energy efficiency of wireless networks by reconfiguring the wireless propagation environment [1]. Specifically, IRS is a planar metasurface consisting of a large number of passive reflecting elements, each of which can reflect the incident signal with a desired phase shift. By adaptively adjusting the propagation of reflected signal, IRS can improve the received signal power via constructive signal combination and destructive interference mitigation at the receivers, thereby enhancing the network performance.

## II. System Model and Problem Formulation

We consider an uplink system where  $K$  single-antenna users transmit signals to a  $M$ -antenna BS with the aid of an IRS. Let  $\mathbf{x}_k$  be the transmit signal, then the received signal of the  $k$ -th user is

$$\mathbf{y}_k = \mathbf{w}_k^H \mathbf{G} \text{diag}(\mathbf{h}_k) \boldsymbol{\theta} \mathbf{x}_k + \mathbf{n}_k, \quad (1)$$

where  $\mathbf{h}_k$  is the channel from the  $k$ -th user to the IRS,  $\mathbf{G}$  is the channel from the IRS to the BS, and  $\mathbf{n}_k$  is the additive Gaussian noise. In this setting, the SINR of the  $k$ -th user is

$$\text{SINR}_k = \frac{|\mathbf{w}_k^H \mathbf{G} \text{diag}(\mathbf{h}_k) \boldsymbol{\theta}|^2}{\sum_{j \neq k} |\mathbf{w}_j^H \mathbf{G} \text{diag}(\mathbf{h}_j) \boldsymbol{\theta}|^2 + \sigma_k^2}. \quad (2)$$

In this paper, our goal is to find out the optimal IRS phase shifts and BS beamforming vectors

minimizing the uplink transmit power. The uplink power minimization problem can be formulated as

$$(\text{P1}) : \min_{\{\boldsymbol{\theta}\}, \{\mathbf{w}_k\}} \sum_{k=1}^K p_k \quad (3a)$$

$$\text{s.t. } \|\mathbf{w}_k\|^2 = 1, \quad (3b)$$

$$|\theta_n| = 1, \quad \forall n \in \mathcal{N}. \quad (3c)$$

Note that due to the non-convexity of the unit-modulus constraint of IRS phase shifts and the unit-norm constraint of BS beamforming vectors, (P1) is non-convex and difficult to solve.

## III. RM-AO Algorithm

In this paper, we propose the RM-AO scheme to solve the problem (P1) by exploiting the Riemannian manifold structures of the unit-modulus IRS phase shifts and the unit-norm BS beamforming vectors [2]. The key idea is to find out the optimal IRS phase shifts and BS beamforming vectors minimizing the uplink transmit power. To be specific, we firstly optimize the IRS phase shifts when the BS beamforming is fixed, and then we optimize the BS beamforming vectors using the obtained IRS phase shifts.

### A. IRS Phase Shifts Optimization

In order to solve (P1), we define (3b) as a complex circle manifold, given by

$$\mathcal{M} = \{\boldsymbol{\theta} \in \mathbb{C}^{N \times 1} : |\theta_n| = 1, \quad \forall n \in \mathcal{N}\}. \quad (4)$$

The tangent space of  $\mathcal{M}$  is

$$\mathcal{T}_{\boldsymbol{\theta}^t} \mathcal{M} = \{\mathbf{u} \in \mathbb{C}^{N \times 1} : \Re\{\mathbf{u} \circ (\boldsymbol{\theta}^t)^*\} = \mathbf{0}_N\}. \quad (5)$$

The Riemannian gradient is the orthogonal proje

ction of the Euclidean gradient onto the tangent space. Therefore, the Riemannian gradient is given by

$$\text{grad}_{\theta^t} f = \nabla_{\theta^t} f - \Re\{(\theta^t)^* \odot \nabla_{\theta^t} f\} \odot \theta^t. \quad (6)$$

Then, the update of the conjugate gradient method on the manifold  $\mathcal{M}$  is given by

$$\mathbf{p}_{\mathcal{M}}^{t+1} = -\text{grad}_{\theta^{t+1}} f_1 + \alpha^t T_{\theta^t \rightarrow \theta^{t+1}}(\mathbf{p}_{\mathcal{M}}^t). \quad (7)$$

After determining the search direction, an operation called retraction is used to find the destination on the manifold. Specifically, the retraction is given by

$$R_{\theta^t}(\beta^t \mathbf{p}_{\mathcal{M}}^t) \triangleq \mathcal{T}_{\theta^t} \mathcal{M} \mapsto \mathcal{M} : \beta^t \mathbf{p}_{\mathcal{M}}^t = \frac{\beta^{t+1} \mathbf{p}_{\mathcal{M}}^{t+1}}{\|\beta^{t+1} \mathbf{p}_{\mathcal{M}}^{t+1}\|}. \quad (8)$$

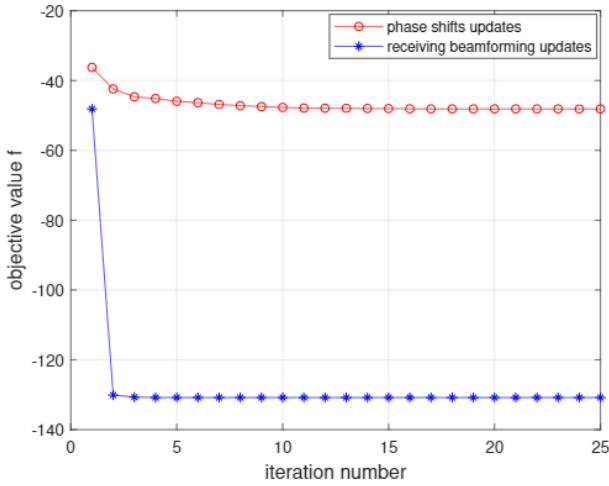
## B. BS Beamforming Optimization

When the IRS phase shift vector is optimized, we update the BS beamforming vector by defining the complex oblique manifold as

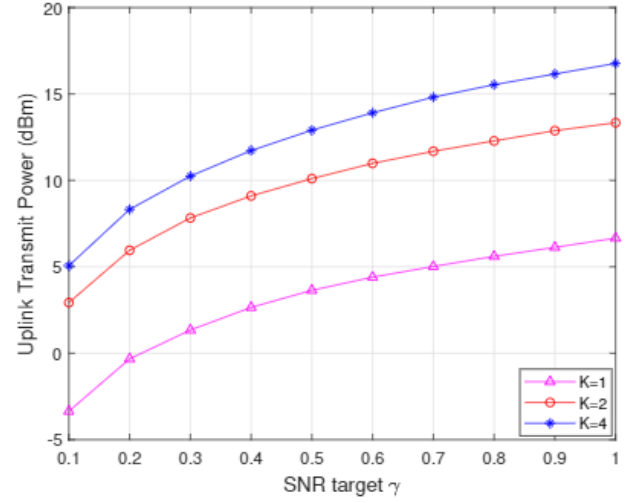
$$\mathcal{N} = \{\mathbf{w} \in \mathbb{C}^{M \times 1} : \|\mathbf{w}\|^2 = 1\}. \quad (9)$$

Then, using the similar update procedures, we optimize the BS beamforming vectors by exploiting the conjugate gradient method.

## IV. Simulation Results



In Fig. 1, we illustrate the convergence curve of the RM-AO algorithm. The results validate the monotonicity and convergence of the proposed algorithm in updating the IRS phase shifts and BS beamforming vectors. We see that at the first few iterations, the updates of IRS phase shifts converges quickly and achieves a local minimal of the objective value when the receiving beamforming is fixed. We also observe that the updates of receiving beamforming vectors converges very fast and achieves the limiting value of the objective function.



In Fig. 2, we plot the uplink transmit power as a function of SINR target. It is observed that the transmit power can be reduced significantly as the SINR target decreases. For example, when the SINR target decreases from 1 to 0.1, we see that RM-AO saves about 12 dBm uplink transmit power at  $K = 4$ ,  $L = 2$ . Similarly, the transmit power can also be reduced as  $K$  decreases. For instance, when  $K$  decreases from 4 to 1, the transmit power decreases more than 10 dBm when the SINR target is 1.

## V. Conclusion

In this paper, we proposed a novel Riemannian manifold-based alternating optimization scheme for the IRS-aided uplink transmission. The key idea of the proposed scheme is to alternatively optimize the phase shifts at the IRS and the receive beamforming vectors at the BS. In order to solve the power minimization problem, we optimize the phase shifts and beamforming vectors over the manifold space in an alternative way. We demonstrate the effectiveness of the proposed scheme in reducing the uplink transmit power.

## ACKNOWLEDGMENT

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## Reference

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