Zero Noise Extrapolation for Quantum State Tomography

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***Abstract*—Quantum states tomography (QST) relies on the accurate measurements of an unknown quantum state to reconstruct its density matrix representation. When the unknown quantum state evolves due to the existence of any noise before the measurement is performed, the reconstructed state will have a low fidelity due to noisy measurement statistics used in its reconstruction. Quantum error mitigation (QEM) techniques are used to suppress errors in noisy quantum devices by performing post-processing on noisy measurement results. Here, prior to state reconstruction, the noisy measurement statistics are mitigated to the ideal ones by implementing one of QEM techniques, e.g., zero noise extrapolation (ZNE). The noise model used in the numerical simulation is the quantum depolarizing channel (QDC). We analyze the best resource allocation for state tomography part and noise extrapolation task.**

I. Introduction

Quantum state tomography (QST) is an important task in quantum information processing to characterize an unknown quantum state and generate its density matrix. We need to characterize a quantum state because it is not an observable. There are two steps in QST. The first one is the measurement of the unknown quantum states under study. Since quantum states collapse after we perform measurement on it, we need to prepare the ensemble of identically quantum states and perform the measurement on each of it to obtain the statistics of the states. The second step is the state reconstruction by using statistical inference method for example maximum likelihood [1]–[3], Bayesian inference [4], linear regression (LR) [5], neural network [6]–[9] and many others based on the statistics of measurement outcomes.

In practice, the measurement apparatus is noisy. Consider performing QST in a noisy environment. Suppose the unknown state is in state ρ. Before measure the state, due to some experimental noise that we can not control, the state turns into ρ^* which is different from the initial state. If we perform measurement on ρ^* and reconstruct the state, the reconstructed state will approximate the state ρ^* instead of ρ. From here we can see that the presence of noise in QST setup lower the fidelity between the reconstructed and actual state.

We need to perform additional tasks to handle the effect of noise in noisy QST. There are two possible ways which are quantum error correction (QEC) and quantum error mitigation (QEM). The QEC techniques make use of a larger number of physical qubits to encode logical qubits to protect them against physical errors. It aims to recover the ideal output of the state [10]. The QEM is used to reduce the errors in noisy quantum experiment [11]. It relies on the classical post processing of the measurement results. Rather than recovering the ideal state, QEM is trying to recover the ideal measurement results. The more sample available when performing QEM, the more accurate the estimation of ideal measurement results will be. To obtain the reconstructed quantum states with high fidelity despite the noisy environment, we perform QEM to mitigate the effects of noise in the measurement results.

One of QEM methods that widely used is zero noise extrapolation (ZNE). The ZNE is based on Richardson extrapolation method to extrapolate the noise-free expectation value of an observable $\boldsymbol{M}$ [12]. Recently, ZNE has been experimentally implemented for finding the ground state energy of H_2 and LiH [13]. In this work, we perform numerical simulations to reconstruct the noise-free quantum states in noisy environment by implementing ZNE order-1 after quantum states are measured. The noisy environment is modeled by quantum depolarizing channel (QDC). The states reconstruction method used is linear regression. We analyzed the best resource allocation for state tomography part and error mitigation task.

The remainder of this paper is organized as follows. In Section II, we review the QST via linear regression, the basic concept of ZNE and method used in conducting the research. Section III shows the numerical simulation results and finally we conclude in Section IV.

II. Methods

In this section, we will review the basic concept needed to perform QST via linear regression and the way to implement ZNE. The qubit estimation by linear regression is obtained by

$$\hat{\boldsymbol{\rho}} = \boldsymbol{I}/2 + \sum_{n=1}^{3} \hat{r}_n \boldsymbol{M}_n, \tag{1}$$

where $\boldsymbol{I}$ is an identity matrix, $\boldsymbol{M}_n$ be the measurement operator such as $\boldsymbol{M}_0 = \boldsymbol{I}$, $\mathrm{tr}(\boldsymbol{M}_n) = 0$ and $\mathrm{tr}(\boldsymbol{M}_m \boldsymbol{M}_n^\dagger) = 2\delta_{mn}$, and $\hat{r}_n$ can be estimated by solving

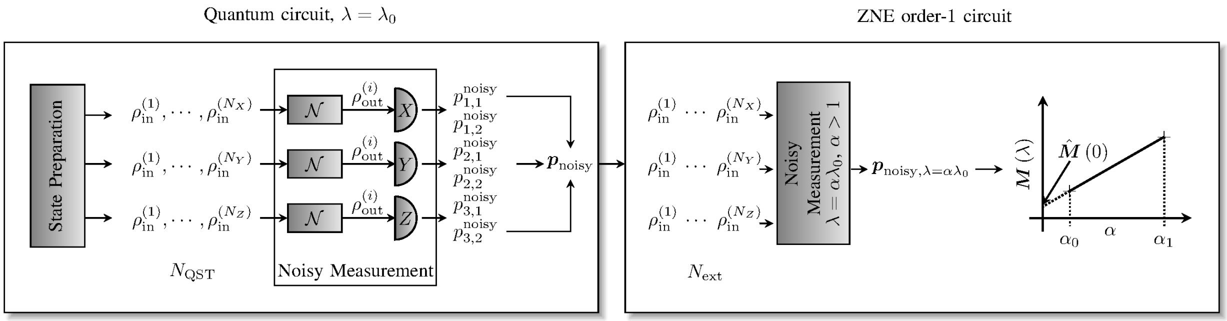

Fig. 1: The illustration of the order-1 ZNE implementation in QST. The unknown quantum states (resources) are allocated for state reconstruction process (N_{QST}) and error mitigation task (N_{ext}). To extrapolate the noise-free expectation value (statistics of measurement outcomes), the state tomography circuit is ran once again with the more stronger noise ($\lambda = \alpha\lambda_0$).

$$\boldsymbol{Y} = \boldsymbol{\Phi r}$$

$$\begin{pmatrix} p_{1,1} - \frac{1}{d} \\ p_{1,2} - \frac{1}{d} \\ \vdots \\ p_{n,d-1} - \frac{1}{d} \\ p_{n,d} - \frac{1}{d} \end{pmatrix} = \begin{pmatrix} \phi^1_{1,1} & \cdots & \phi^1_{d^2-1,1} \\ \phi^2_{1,1} & \cdots & \phi^2_{d^2-1,1} \\ \vdots & \ddots & \vdots \\ \phi^{d-1}_{1,n} & \cdots & \phi^{d-1}_{d^2-1,n} \\ \phi^{d}_{1,n} & \cdots & \phi^{d}_{d^2-1,n} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_{d^2-1} \end{pmatrix} \tag{2}$$

where $p_{n,i}$ is the ideal probability of obtaining measurement result corresponding to M_n and $\phi^i_{k,n} = \mathrm{tr}\left(|u_{n,i}\rangle \langle u_{n,i}| M_k\right)$ or the eigenvalue corresponding to M_k and $|u_{n,i}\rangle$.

The illustration of ZNE implementation in QST can be seen in Fig. 1. To reconstruct the noise-free quantum states with ZNE, the available resources (ensemble of the unknown quantum states) are divided into N_{QST} to obtain the noisy state information from noisy measurement where noise parameter λ is λ_0 and N_{ext} to obtain the statistics of the state with $\lambda = \alpha\lambda_0$, $\alpha > 1$ where α denotes the noise amplified parameter. After we run both circuit, we obtain two measurement outcomes statistics, $M(\lambda_0)$ and $M(\alpha_1\lambda_0)$. By ZNE order-1, the noise-free statistics of our states can be calculated as,

$$\boldsymbol{M}(0) = \frac{\alpha_1}{\alpha_1 - 1}\boldsymbol{M}(\lambda_0) + \frac{1}{1-\alpha_1}\boldsymbol{M}(\alpha_1\lambda_0). \tag{3}$$

To reconstruct the noise-free quantum states, the elements of $\boldsymbol{M}(0)$ are used to obtain $\hat{r}_n$ by eq. 2 then the $\hat{r}_n$ is substituted to eq. 1.

III. Numerical Results

We perform numerical simulation on one qubit system with scheme explained in the previous section. To quantify how close the actual state and its estimation, we calculate the fidelity of both state $F(\rho, \hat{\rho}) = \mathrm{tr}\left(\sqrt{\sqrt{\rho}\hat{\rho}\sqrt{\rho}}\right)$. We then calculate the average log infidelity $\log_{10}(1 - F(\rho, \hat{\rho}))$ of 1×10^4 mixed states. The resources available is 1×10^5 states. We divided the resources into N_{QST} and N_{ext} with some different allocation ratio and we will observe which scenario works the best in reconstructing the quantum states.

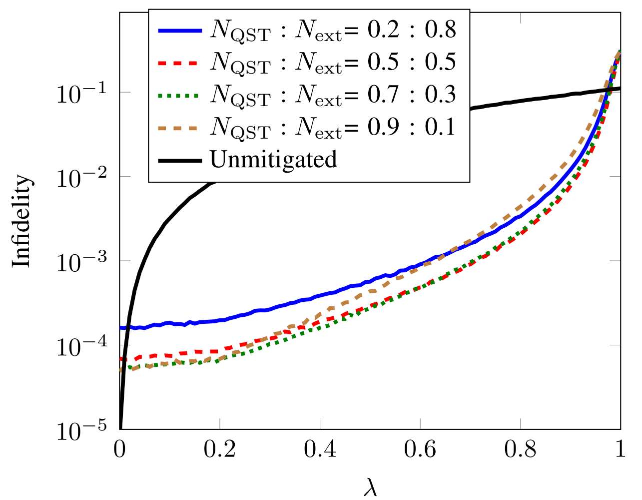

Fig. 2: Average infidelity of reconstructed and actual quantum states of 1×10^5 with $\alpha = 5$ for various resource allocation scheme as a function of QDC noise parameter.

Fig. 2 shows the average infidelity of reconstructed and actual state as a function of QDC noise parameter. Since the infidelity represents how far the actual and reconstructed state is, the smaller the value the better the result we get. We can see that if we use $\boldsymbol{M}(\lambda_0)$ to reconstruct ρ without being mitigated first, the reconstructed state has low fidelity (black line) compare to states reconstructed with $\boldsymbol{M}(0)$. From the figure we observe that if we increase the allocation ratio of N_{QST} from 0.2 to 0.7, we can increase the fidelity but when we increase the resources ratio to 0.9 the result get worse. So when ZNE is implemented in noisy QST, from the simulation result the best ratio for N_{QST} and N_{ext} is 0.7 by 0.3.

IV. Conclusion

We studied about the effect of ZNE implementation in quantum state tomography. We found that by implementing ZNE to mitigate the noisy measurement results we can improve the fidelity of the true state and reconstructed state. The best resource allocation ratio for N_{QST} and N_{ext} we found by numerical simulation is 0.7 by 0.3. The possible future work is investigating the effect of other QEM techniques in QST and experimentally implement QEM for QST in IBMQ computer.

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