Statistical Quantum Federated Learning for NOMA Power Allocation

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Abstract

This study employs the statistical Quantum Federated Learning (sQFL) to optimize NOMA power allocation. Compared to the existing Federated Learning (FL), sQFL does not require other edges to perform neural network inferences. The other edge only required to transmit the statistical information to the cloud.

Keywords: Quantum neural networks, quantum federated learning, 6G, wireless communications

To maintain users' data privacy and distribute the computational load, federated learning (FL) can be employed to optimize the future wireless network [1]. On the other hand, quantum computation has been studied to enhance the processing of machine learning [2]. The existing federated learning approach continuously includes the neural network of the "informant" in the training process [3].[1] To reduce the burden of the "informant" (in terms of computation and communication complexity), this paper proposes a statistical quantum federated learning approach (sQFL). In the sQFL scheme, the "informant" is only required to send the dataset statistics (without performing neural network inference).

Notations: Let $\mathbf{H}$, $\mathbf{C}_{\mathrm{Z}}$, $\mathbf{C}_{\mathrm{X}}$, and $\mathbf{R}_{\mathrm{Z}}$ be the operation of Hadamard gate, controlled Z gate, controlled X gate, and rotation on the Z axis, respectively. Quantum measurement is denoted as $\mathbf{M}(\cdot)$. Let $x \sim \mathcal{CN}(\mu, \sigma^2)$, where μ and σ^2 denotes the mean and the variance, respectively, be a complex Gaussian distribution. The notation of $\otimes$ expresses Knocker product. The notations of $\mathbb{R}$ and $\mathbb{C}$ denote the real and complex numbers, respectively.

1. System Model

Consider the mth radio access network (RAN) with a radio access point (RAP) for downlink (DL) non-orthogonal multiple access (NOMA). Consider $\mathrm{u}_{\mathrm{str}}^{[m]}$ and $\mathrm{u}_{\mathrm{weak}}^{[m]}$ as the user with a stronger and a weaker channel gain, accordingly. Let $d_{\mathrm{str}}^{[m]}$ and $d_{\mathrm{weak}}^{[m]}$ be the normalized distance values between the AP and $\mathrm{u}_{\mathrm{str}}^{[m]}$ and $\mathrm{u}_{\mathrm{weak}}^{[m]}$, respectively. Considering ρ as the transmit signal-to-noise ratio (SNR), the channel coefficients of $\mathrm{u}_{\mathrm{str}}^{[m]}$ and $\mathrm{u}_{\mathrm{weak}}^{[m]}$ are modeled as $h_{\mathrm{str}}^{[m]} \sim \mathcal{CN}(0, (d_{\mathrm{str}}^{[m]})^{-\vartheta})$ and $h_{\mathrm{weak}}^{[m]} \sim \mathcal{CN}(0, (d_{\mathrm{weak}}^{[m]})^{-\vartheta})$, respectively, where ϑ is the pathloss exponent [4]. The receive signal-to-interference-plus-noise-ratio (SINR) for $\mathrm{u}_{\mathrm{str}}^{[m]}$ and $\mathrm{u}_{\mathrm{weak}}^{[m]}$ can be expressed as $\gamma_{m,\mathrm{str}}^{\mathrm{NOMA}} = |h_{\mathrm{str}}^{[m]}|^2 \rho \lambda_{\mathrm{str}}^{[m]}$ and $\gamma_{n,\mathrm{weak}}^{\mathrm{NOMA}} = \frac{|h_{\mathrm{weak}}^{[m]}|^2 \rho \lambda_{\mathrm{weak}}^{[m]}}{|h_{\mathrm{weak}}^{[m]}|^2 \rho \lambda_{\mathrm{str}}^{[m]} + 1}$, respectively [5]. Subsequently, the sum rate of the NOMA users can be expressed as $R_{m,\mathrm{sum}}^{\mathrm{NOMA}} = R_{m,\mathrm{str}}^{\mathrm{NOMA}} + R_{m,\mathrm{weak}}^{\mathrm{NOMA}}$, where the achievable rate of $\mathrm{u}_{\mathrm{str}}^{[m]}$ and $\mathrm{u}_{\mathrm{weak}}^{[m]}$ is defined as $R_{m,\mathrm{str}}^{\mathrm{NOMA}} = \log_2(1 + \gamma_{m,\mathrm{str}}^{\mathrm{NOMA}})$ and $R_{m,\mathrm{weak}}^{\mathrm{NOMA}} = \log_2(1 + \gamma_{m,\mathrm{weak}}^{\mathrm{NOMA}})$, accordingly. Finally, the objective of the NOMA power allocation optimization can be expressed as [6]:

$$\underset{\lambda_{\mathrm{str}}^{[m]}}{\text{maximize}} \quad R_{m,\mathrm{sum}}^{\mathrm{NOMA}}(\lambda_{\mathrm{str}}^{[m]}, \lambda_{\mathrm{weak}}^{[m]} = 1 - \lambda_{\mathrm{str}}^{[m]}) \tag{1a}$$

$$\text{subject to} \quad R_{m,\mathrm{str}}^{\mathrm{NOMA}} \geq R_{m,\mathrm{str}}^{\mathrm{OMA}}, R_{m,\mathrm{weak}}^{\mathrm{NOMA}} \geq R_{m,\mathrm{weak}}^{\mathrm{OMA}}, \tag{1b}$$

$$0 \leq \lambda_{n,\mathrm{str}}^{\mathrm{NOMA}} \leq 1, \tag{1c}$$

where the achievable rate of $\mathrm{u}_{\mathrm{str}}^{[m]}$ and $\mathrm{u}_{\mathrm{weak}}^{[m]}$ by utilizing orthogonal multiple access (OMA) is denoted as $R_{m,\mathrm{str}}^{\mathrm{OMA}}$ and $R_{m,\mathrm{weak}}^{\mathrm{OMA}}$, respectively.

2. Proposed Scheme

The proposed sQFL (Fig. 1) can be described as follows. For this initial study, the presented scheme only considers one edge (therefore, no aggregation required).[2]

[1]For the sake of simplicity, this paper refers to the computing unit that processes the local dataset as the "informant." Moreover, let us refers to the computing unit that obtains the information from "informants" as "recipients."

[2]This study considers that the quantum computation can be employed to complement the future network function. Moreover, this

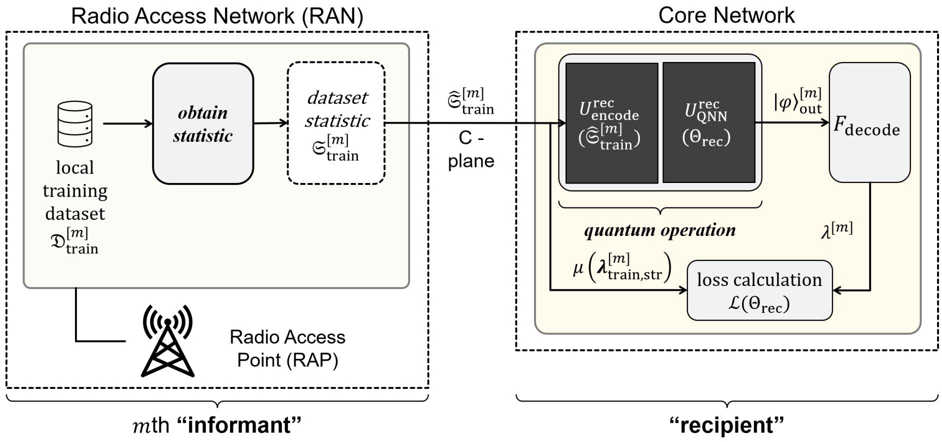

Figure 1. The proposed sQFL scheme.

2.1. Dataset Statistics

Consider N_{info} as the number of the informants. Let $N^{[m]}_{\text{train}}$ be the number of training data for the mth informant, $m \in \{1,\ldots,N_{\text{info}}\}$. Considering $k \triangleq \{h^{[m,j]}_{\text{train,str}}, h^{[m,j]}_{\text{train,weak}}, \lambda^{[m,j]}_{\text{train,str}}\}$ and $K \triangleq \{\boldsymbol{h}^{[n]}_{\text{train,str}}, \boldsymbol{h}^{[n]}_{\text{train,weak}}, \boldsymbol{\lambda}^{[n]}_{\text{train,str}}\}$, the mean and standard deviation of the strong user channel, weak user channel, and strong user power allocation of the mth informant can be expressed as: $\mu(K) = \frac{1}{N^{[m]}_{\text{train}}}\sum_{j=1}^{N^{[m]}_{\text{train}}} k$ and $\sigma(K) = \sqrt{\frac{1}{N^{[m]}_{\text{train}}}\sum_{j=1}^{N^{[m]}_{\text{train}}} \lambda^{[m,j]}_{\text{train,str}} - \mu(K)^2}$, respectively. The confidence interval is denoted as $\varepsilon(\boldsymbol{\lambda}^{[m]}_{\text{train,str}}) = \mu(\boldsymbol{\lambda}^{[m]}_{\text{train,str}}) \pm z_{\text{value}}\frac{\sigma(\boldsymbol{\lambda}^{[m]}_{\text{train,str}})}{\sqrt{N^{[m]}_{\text{train}}}}$, where z_{value} denotes the z-value with respect to a determined confidence level.[3] The dataset statistics of the mth edge can be concatenated by the recipient as a tuple of

$$\begin{aligned}\mathfrak{S}^{[m]}_{\text{train}} =& \Big[\varepsilon(\boldsymbol{\lambda}^{[m]}_{\text{train,str}}); \{\mu(\boldsymbol{h}^{[m]}_{\text{train,str}}), \sigma(\boldsymbol{h}^{[m]}_{\text{train,str}})\}; \\ & \{\mu(\boldsymbol{h}^{[m]}_{\text{train,weak}}), \sigma(\boldsymbol{h}^{[m]}_{\text{train,weak}})\}; \{\mu(\boldsymbol{\lambda}^{[m]}_{\text{train,str}}), \sigma(\boldsymbol{\lambda}^{[m]}_{\text{train,str}})\}\Big].\end{aligned} \tag{2}$$

2.2. QNN Circuit

The considered quantum neural network (QNN) circuit of the recipient can be described as follows. The encoding operation, which is based on [8], is expressed as

$$U^{\text{rec}}_{\text{encode}}(\mathfrak{S}^{[m]}_{\text{train}}) = \bigotimes_{n=1}^{N_{\text{info}}} \bigotimes_{i=1}^{\left|\mathfrak{S}^{[N_{\text{info}}]}_{\text{train}}\right|} \mathbf{R}_z\Big(\underbrace{\tanh\big(x_i^{[m]}\big)}_{\text{pre-processing}}\Big), \quad x_i^{[m]} \in \mathfrak{S}^{[N_{\text{info}}]}_{\text{train}}. \tag{3}$$

where $x^{[m]}$ indicates the input values from nth informant, i.e., elements of $\mathfrak{S}^{[m]}_{\text{input}}$. Let $\left|\mathfrak{S}^{[N_{\text{info}}]}_{\text{train}}\right|$ be the number of elements of $\mathfrak{S}^{[m]}_{\text{train}}$. Consider $\tanh(\cdot)$ function as the pre-processing operation. Let Θ_{rec} be the set of weights. Considering N_{layer} and $N^{[l]}_{\text{neuron}}$ as the number of layers and neuron, respectively, the quantum dense layers (based on [9]) can be presented as

$$\begin{aligned} U^{\text{rec}}_{\text{QNN}}(\Theta_{\text{rec}}) =& \Big(\prod_{j=1}^{N^{[N_{\text{layer}}]}_{\text{neuron}}-1} \mathbf{C}_Z(q_{N_{\text{layer}},j+1}; q_{N_{\text{layer}},j})\Big) U_A \\ & \Big(\bigotimes_{l=1}^{N_{\text{layer}}} \bigotimes_{j=1}^{N^{[l]}_{\text{neuron}}} \mathbf{R}_z(\theta^{\text{rec}}_{l,j}),\Big), \\ U_A \triangleq & \begin{cases} \prod_{j=1}^{N^{[l]}_{\text{neuron}}-1} \mathbf{C}_Z(q_{2,1}; q_{1,j}), \text{ for } l=1, \\ \prod_{l=1}^{N_{\text{layer}}-1} \mathbf{C}_X(q_{l+1,1}; q_{l,N^{[l]}_{\text{neuron}}})\Big(\prod_{j=1}^{N^{[l]}_{\text{neuron}}-1} \mathbf{C}_Z(q_{l,j+1}; q_{l,j})\Big), \\ \qquad \text{for } 2 < l < N_{\text{layer}}, \end{cases} \end{aligned} \tag{4}$$

where $\theta^{\text{rec}}_{l,j} \in \Theta_{\text{rec}}$ is the weight. Considering $|\psi\rangle^{[m]}_{\text{out},r}$ as the output state of $U^{\text{rec}}_{\text{QNN}}(\Theta_{\text{rec}})$ for the rth quantum measurement, the decoding function can be expressed as $F_{\text{decode}}(|\psi\rangle^{[m]}_{\text{out}}) = \frac{1}{N_{\text{measure}}}\sum_{r=1}^{N_{\text{measure}}} \mathbf{M}(|\psi\rangle^{[m]}_{\text{out},r})$, where N_{measure} is the number of quantum measurements and

paper assumes that the QNN data can be communicated via the control plane (C-plane) [7].

[3] For instance, $z_{\text{value}} = 1.96$ is employed for 95% confidence level.

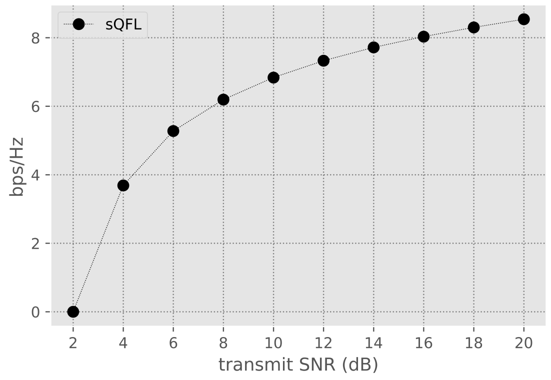

Figure 2. Initial simulation result of sQFL for NOMA power allocation.

$|\psi\rangle^{[m]}_{\text{out}} = \{|\psi\rangle^{[m]}_{\text{out},1}, \ldots, |\psi\rangle^{[m]}_{\text{out},N_{\text{measure}}}\}$. Subsequently, consider $\lambda^{[m]}_{\text{str}} = F_{\text{decode}}(|\psi\rangle^{[m]}_{\text{out}})$.

3. Result

A simply iterative training (Algorithm 1) is employed for this initial study. Let $\mathcal{W}$ as the set of weight options, where the ith weight options ($w \in \mathcal{W}$) can be expressed as $\omega_i = -2\pi + (i-1)\frac{2\pi}{N_\omega}$, where $\omega_i \leq 2\pi$ and $l \geq 1$.

Algorithm 1 Training

Output: Optimized Θ_{rec}.

Initialization:

Set all qubits in $U^{\text{rec}}_{\text{encode}}$, $U^{\text{rec}}_{\text{QNN}}$ to $|0\rangle$. Set all weights in Θ_{rec} to 0.

Training:

Calculate $\mathfrak{S}^{[m]}_{\text{train}}$ (Section. 2.1).

for each weight in Θ_{rec} **do**

  for all weight options **do**

    Consecutively, perform $U^{\text{rec}}_{\text{encode}}$, $U^{\text{rec}}_{\text{QNN}}$, and F_{decode} to obtain $\lambda^{[n]}_{\text{str}}$ (Section 2.2).

    Calculate loss $\mathcal{L}(\Theta_{\text{rec}}) = |\lambda^{[m]}_{\text{str}} - \mu(\lambda^{[m]}_{\text{train,str}})|^2$.

  end for

  Determine $\theta^{\text{rec}}_{l,j} = \arg\min_{\theta^{\text{rec}}_{l,j} \in \Theta_{\text{rec}}} \mathcal{L}(\Theta_{\text{rec}})$.

end for=0

The quantum operations ($U^{\text{rec}}_{\text{encode}}$ and $U^{\text{rec}}_{\text{QNN}}$) were performed using IBM Q (using IBM Qiskit [10]), where $N_{\text{measure}} = 1024$ were considered. The simulation parameters are considered as follows: $\vartheta = 2$ and $N_{\text{data}} = 100$. For each jth training data, $j \in \{1, \ldots, N^{[m]}_{\text{train}}\}$, the power allocation coefficient is obtained as $\lambda^{[m,j]}_{\text{train,str}} = \frac{\sqrt{1+(\|h^{[m,j]}_{\text{train,str}}\|^2\rho)}-1}{\|h^{[m,j]}_{\text{train,str}}\|^2\rho}$, [6] (further described in **Appendix**). For simplicity, this paper assumes $\rho = 10$ dB for all training data. The average result of Monte-Carlo simulation (10^4 trials) is presented in Fig. 2. The considered quantum circuit for simulation is presented in **Appendix**.

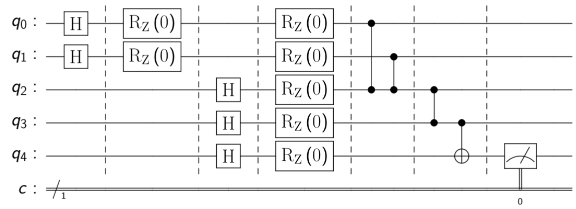

Figure 3. QNN circuit for simulation (includes encoding operation).

4. Future Work

This paper investigated the usage of the presented qSFL for optimizing power allocation in NOMA. The future work can consider other data statistics.

Appendix

For training data, the calculation of $\lambda^{[m,j]}_{\text{train,str}}$, i.e., the optimal power allocation for $\mathrm{u}^{[m]}_{\text{str}}$, is based on [6]. Specifically, considering the constraints of $\lambda^{[m,j]}_{\text{train,str}} \leq \frac{\sqrt{1+(\|h^{[m,j]}_{\text{train,str}}\|^2\rho)}-1}{\|h^{[m,j]}_{\text{train,str}}\|^2\rho}$ and $\lambda^{[m,j]}_{\text{train,str}} \geq \frac{\sqrt{1+(\|h^{[m,j]}_{\text{train,str}}\|^2\rho)}-1}{\|h^{[m,j]}_{\text{train,str}}\|^2\rho}$, the following condition requires to be satisfied $\lambda^{[m,j]}_{\text{train,str}} = \frac{\sqrt{1+(\|h^{[m,j]}_{\text{train,str}}\|^2\rho)}-1}{\|h^{[m,j]}_{\text{train,str}}\|^2\rho}$.

For simulation, the quantum circuit in Fig. 3 was employed. The QNN parameters are presented as follows: $N_{\text{layer}} = 1$, $N^{[1]}_{\text{neuron}} = 2$, $N^{[2]}_{\text{neuron}} = 2$ and $N^{[3]}_{\text{neuron}} = 1$.

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