

Low-Complexity Transmit Power Design for Prioritized Wireless Mutual Broadcast

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ABSTRACT

Wireless mutual broadcast is essential for proximity-aware services in dynamic networks. This study proposes a low-complexity transmit power design method to improve the performance of a specific node group while limiting overall performance loss, in particular, with closed-form solutions for a path loss exponent of four. Numerical results demonstrate its accuracy and usefulness.

Key Words : Heterogeneous networks, neighbor discovery, transmit power control, D2D, ad hoc network

I. Introduction

As 6G and AI advance, proximity-aware services enable localized tasks like neighbor discovery and safety messaging in IoT and vehicle networks^[1]. Wireless mutual broadcast (WMB)^[2-4] supports these services by broadcasting presence and data to nearby nodes. WMB for devices with heterogeneous attributes^[5-7] is crucial for diverse service needs. Some node groups, like safety-critical systems, require higher broadcast message (BM) success rates, but prioritizing them may degrade others' performance^[7]. This study proposes a low-complexity transmit (Tx) power configuration to improve prioritized group performance while maintaining network efficiency.

Stochastic geometry has been widely applied to analyze RA-WMB performance in spatial configurations like HPPP^[2], repulsive^[3], and clustered^[4] node distributions. However, these studies did not address

heterogeneous nodes. Heterogeneous RA-WMB networks have been explored for diverse node characteristics. [5] analyzed local broadcasting with varying Tx power levels but lacked joint Tx power optimization. [6] studied networks with half-and full-duplex nodes, offering spatial insights but without Tx power control or prioritization. [7] jointly optimized Tx power for two node groups, balancing one group's performance with system-wide degradation but at high computational complexity.

This paper proposes: (i) A joint Tx power optimization method to prioritize performances across node groups with much lower complexity than [7]; (ii) Closed-form optimal Tx powers for a path loss exponent (PLE) of four.

Notations. $\mathbb{E}[\mathcal{X}]$, $\mathbb{P}[E]$, and x^* are the expected value of $\mathcal{X}$ with respect to X , the probability of event E , and the optimal value of x $\mathbb{1}[Y=y]$ is the indicator function, equal to 1 if $Y=y$ and 0 otherwise.

II. Performance Models for RA-WMB Using Heterogeneous Transmit Power

This paper investigates heterogeneous RA-WMB networks with nodes using different Tx powers. HPPP simplifies analysis for slotted Aloha, while advanced access schemes and clustered distributions, exhibiting trends similar to HPPP^[3,4], are left for future work. In this HPPP, node i and its location are denoted by X_i , with a Tx power value of $m_i \in \{p_1, \dots, p_G\}$ determined by priority or service requirements. Priority-based Tx power can be managed via Tx power control protocols, e.g., periodic broadcasts of Tx power policies by header nodes, including the Tx power derived in this study. The nodes and their Tx power values are collectively modeled as the marked HPPP $\hat{\Phi} \triangleq \{(X_i, m_i)\}$. Each node transmits its BM with TxPr ν , randomly selecting one of K orthogonal resource blocks (RBs), and receives with probability $1 - \nu$. This study focuses on varying Tx power levels with a common TxPr ν , leaving TxPr variation for

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future work. Wireless channels assume path loss with PLE α and Rayleigh fading, as Nakagami- m has minimal impact on overall trends^[2]. Node i 's transceiving status T_i is 1 for transmission and 0 for reception. Interference status U_{ij} is 1 if node j transmits on the same RB as node i , and 0 otherwise. For the typical node X_0 at the origin decoding a BM from X_i with Tx power m_i , the SINR is $\Xi_i = \frac{m_i h_i |X_i|^{-\alpha}}{I(X_i) + \sigma^2}$ where $I(X_i) = \sum_{j: (X_j, m_j) \in \hat{\Phi} \setminus \{(X_i, m_i)\}} \mathbb{1}[U_{ij} = 1] m_j h_j |X_j|^{-\alpha}$, and $\sigma^2 = \frac{\tilde{\sigma}^2}{A}$. $\tilde{\sigma}^2$ is noise power, A is path loss gain at unit distance, h_i is the Rayleigh fading gain, and $|X_j|$ is the distance of X_j from X_0 . Decoding succeeds if $\Xi_i > \xi$.

Nodes with similar attributes are grouped, with each group assigned a Tx power from $\{p_1, \dots, p_G\}$, and nodes in a group use the same Tx power.

This Tx power configuration is expressed as $\Omega \triangleq \{(p_1, \rho_1), \dots, (p_G, \rho_G)\}$, where $\rho_g \triangleq \mathbb{P}[m_i = p_g]$ for $(X_i, m_i) \in \hat{\Phi}$. Given Ω and ν , the performance of group g , $S_g(\Omega, \nu)$, is defined as

$$\mathbb{E} \left[\mathbb{1}[T_0 = 0] \sum_{i: (X_i, m_i) \in \hat{\Phi}} \mathbb{1}[T_i = 1, m_i = p_g, \Xi_i > \xi] \right] \quad (1)$$

which means the average number of BMs that the typical node successfully receives from nodes in group g . The overall performance is given by $S(\Omega, \nu) \triangleq \sum_{g=1}^G S_g(\Omega, \nu)$, representing the average number of BMs received from all groups' nodes. To compare with uniform Tx power, the performance under a common power p is defined as $S_o(p, \nu) \triangleq S(p, 1, \nu)$. [7] concisely expressed S and S_g in terms of S_o , as follows.

Lemma 1 Given Ω and ν , when $\Delta(\alpha) \triangleq \frac{\sin(2\pi/\alpha)}{2\pi/\alpha}$,

$$S(\Omega, \nu) = S_o(\bar{p}_o, \nu), \text{ where } \bar{p}_o \triangleq \mathbb{E}[P^{2/\alpha}]^{\alpha/2}, \quad (2)$$

$$S_o(p, \nu) = \pi \lambda \nu (1 - \nu) p \int_0^\infty e^{-\xi \sigma^2 u^{\alpha/2} - \pi \frac{\lambda \xi^{2/\alpha}}{K \Delta(\alpha)} p \nu u} du \quad (3)$$

$$\text{and } S_g(\Omega, \nu) = \frac{p_g p_g^{2/\alpha}}{\mathbb{E}[P^{2/\alpha}]} S(\Omega, \nu). \quad (4)$$

Further, when $\Theta(\Omega) \triangleq \frac{\max_{0 < \nu < 1} S(\Omega, \nu)}{\max_{0 < \nu < 1} S_o(\bar{p}, \nu)}$, $\Theta(\Omega) < 1$, $\lim_{K \rightarrow \infty} \Theta(\Omega) = \frac{\mathbb{E}[P^{2/\alpha}]}{\mathbb{E}[P]^{2/\alpha}} < 1$, and $\lim_{\sigma^2 \rightarrow 0} \Theta(\Omega) = 1$.

Proof In [7], see Theorem 1 for S , Corollary 2 for S_g , and Corollary 3 for Θ .

III. Transmit Power Design for Performance Prioritization

This section jointly optimizes Tx power for node groups to enhance the performance of a prioritized group (i.e., group 1) while maintaining overall performance above a certain level. Performance prioritization reduces overall performance S because $\Theta(\Omega) < 1$ from Lemma 1. For meaningful prioritization, $p_1 > \bar{p}$ is assumed, where $\bar{p} = \mathbb{E}[P] = \sum_{i=1}^G \rho_i p_i$, with P as the random variable representing node group Tx powers.

[7] focused on two node groups and formulated the problem with the overall performance loss requirement $\Theta(\Omega) \geq \eta$ for $0 < \eta < 1$, defined in Lemma 1, like

$$\begin{aligned} & \underset{p_1, p_2 \geq 0, \rho_1 p_1 + \rho_2 p_2 = \bar{p}}{\text{maximize}} && (\rho_1 p_1^{2/\alpha}) / (\rho_2 p_2^{2/\alpha}) \quad (5a) \\ & \text{subject to} && \Theta(\Omega) \geq \eta. \quad (5b) \end{aligned}$$

In this problem, the computation of $\Theta(\Omega)$ requires complex cascaded iterations evaluating the numerical integrals in S and S_o , finding their maximum TxPr values using the golden section algorithm, and applying the bisection method to the overall process for the final solution. This Θ reaches its minimum in the coverage-limited scenario and gradually increases with interference, converging to one in an interference-limited environment^[7]. To reduce the complexity, this paper replaces $\Theta(\Omega)$ with its lower bound or worst-case value, $\lim_{K \rightarrow \infty} \Theta(\Omega) = \frac{\mathbb{E}[P^{2/\alpha}]}{\mathbb{E}[P]^{2/\alpha}}$, like

$$\underset{p_1, \dots, p_G \geq 0, \mathbb{E}[P] = \bar{p}}{\text{maximize}} && (\rho_1 p_1^{2/\alpha}) / \mathbb{E}[P^{2/\alpha}] \quad (6a)$$

$$\text{subject to} && \mathbb{E}[P^{2/\alpha}] \geq \eta \mathbb{E}[P]^{2/\alpha} \quad (6b)$$

Unlike Problem (5) considering only two groups, the proposed Problem (6) generalizes to any number

of groups. And, it replaces complex $\max_{0 < v < 1} \mathcal{S}(\Omega, v)$ and $\max_{0 < v < 1} \mathcal{S}_o(\bar{p}, v)$ with simpler $\mathbb{E}[P^{2/\alpha}]$ and $\mathbb{E}[P]^{2/\alpha}$, significantly reducing complexity.

Theorem 1 Problem (6) is equivalent to the following problem with $p_2, \dots, p_G$ assigned equally to p_o :

$$\underset{p_1 \geq 0, \rho_1 p_1 + \rho_o p_o = \bar{p}}{\text{maximize}} \quad p_1 / p_o \quad (7a)$$

$$\text{subject to} \quad \rho_1 p_1^{2/\alpha} + \rho_o p_o^{2/\alpha} \geq \eta \bar{p}^{2/\alpha}, \quad (7b)$$

where $\rho_o \triangleq 1 - \rho_1$. Then, for Problem (7), if $\eta > \rho_1^{1-2/\alpha} p_1^*$ is the unique solution of $f(x) = 0$ and $p_2^* = \dots = p_G^* = \bar{p} - \rho_1 p_1^* / \rho_o$. Here, $f_p(x) \triangleq \rho_o^{1-\alpha/2} (\bar{p} - \rho_1 x)^{2/\alpha} + \rho_1 x^{2/\alpha} - \eta \bar{p}^{2/\alpha}$ and $f_p(x)$ decreases monotonically in x .

Otherwise, $p_1^* = \bar{p} / \rho_1$ and $p_2^* = \dots = p_G^* = 0$.

Proof Assume p_1 is fixed. Then, maximizing (6a) reduces to minimizing $\sum_{i=2}^G \rho_i p_i^{2/\alpha}$. By the arithmetic-geometric mean inequality, $\sum_{i=2}^G \rho_i p_i^{2/\alpha} \geq (\sum_{i=2}^G \rho_i) \prod_{i=2}^G (p_i^{2/\alpha})^{\rho_i / \sum_{i=2}^G \rho_i}$, with equality when $p_2 = \dots = p_G$. Thus, minimizing $\sum_{i=2}^G \rho_i p_i^{2/\alpha}$ ensures that $p_2 = \dots = p_G$ are set to the same value, p_o . As a result, maximizing (6a) for all $p_1, \dots, p_G$ becomes equivalent to minimizing $\frac{\rho_o p_o^{2/\alpha}}{\rho_1 p_1^{2/\alpha}}$, or equivalently, maximizing $\frac{p_1}{p_o}$.

Hence, Problems (6) and (7) are equivalent.

From [7, Lemma 1], $\rho_1 p_1^{2/\alpha} + \rho_o p_o^{2/\alpha}$ decreases as p_1 increases. (7a) increases with p_1 . Thus, p_1 can grow until the equality in (7b) is met. Because $p_1 \leq \bar{p} / \rho_1$ from $\mathbb{E}[P] = \bar{p}$, if (7b) holds at $p_1 = \bar{p} / \rho_1$, then $p_1^* = \bar{p} / \rho_1$. Otherwise, i.e., if $\rho_1 (\bar{p} / \rho_1)^{2/\alpha} < \eta \bar{p}^{2/\alpha}$ (or $\eta > \rho_1^{1-2/\alpha}$), p_1 must satisfy the equality in (7b). Combining this with $\mathbb{E}[P] = \bar{p}$ leads to $f_p(p_1^*) = 0$, where $f_p(x)$ monotonically decreases. Thus, $f_p(x) = 0$ has a unique solution, which is p_1^* . The corresponding p_o^* can be derived from $\rho_1 p_1 + \rho_o p_o = \bar{p}$. ■

Theorem 1 demonstrates that optimizing two node groups is equivalent to considering any number of groups, with the optimal Tx power easily found using

the bisection method on the simple $f_p(x) = 0$.

Corollary 1 When $\alpha = 4$ and $p_1 > \bar{p}$, if $\eta > \sqrt{\rho_1}$,

$$p_1^* = \bar{p} \left(\eta + \sqrt{\frac{1-\rho_1}{\rho_1} (1-\eta^2)} \right)^2 \quad (8)$$

$$\text{If } \eta \leq \sqrt{\rho_1}, \quad p_1^* = \bar{p} / \rho_1, \quad p_2^* = \dots = p_G^* = \frac{\bar{p} - \rho_1 p_1^*}{\rho_o}$$

Proof Applying the results of Theorem 1 for $\alpha = 4$ and considering $p_1 > \bar{p}$, the results are derived. ■

IV. Numerical Results and Discussions

This section evaluates the Tx power design from Section III using the model in Section II and parameters in Table 1, with $\mathbb{E}[P]$ fixed at $\bar{p}$. From Theorem 1, the Tx power design of (6) for any number of node groups is equivalent to (7) with $G = 2$; thus, the performance is evaluated with $G = 2$.

Fig. 1 presents the overall and group-specific performance as a function of p_1 / p_o , with $\mathbb{E}[P] = \bar{p}$ held constant while varying p_1 and p_o . A coverage-limited environment without interference is applied to validate the basic properties of prioritized WMB and the accuracy of Table 1. Evaluation Parameters for RA-WMB.

Parameters	Values
S : Average number of successfully received BMs from all nodes per node	
S_g : S for node group g with transmit (Tx) power p_g	
S_o : S with common Tx power	
Θ : Relative performance, $\max_{0 < v < 1} \mathcal{S}(\Omega, v) / \max_{0 < v < 1} \mathcal{S}_o(\bar{p}, v)$	
Ω : Set of (p_g, ρ_g) denoting Tx power configuration	
λ : Total node density	20 km ⁻²
λ_g : node density of node group g	-
$\bar{p}$: Average Tx power	100 mW
p_g : Tx power of node group g	-
ρ_g : Portion of nodes with Tx power p_g	-
v : Transmission probability (TxPr)	$0 < v < 1$
ξ : SINR threshold for successful BM	0 dB
A : Path loss gain at a unit distance	-44.48 dB @ 4 GHz & 1 m
K : Number of orthogonal RBs	1, possible to be more
α : Path loss exponent (PLE)	4, 4.5
σ^2 : Noise power normalized by A	-73.95 dBm for 360 kHz RB
η : maximum performance loss ratio	$0 < \eta < 1$

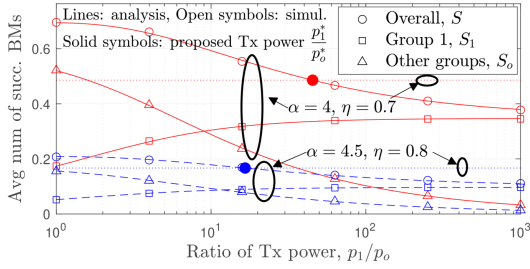


Fig. 1. Tx power design for prioritization in interference-free scenarios. [$\lambda_g = 1 : 3$]

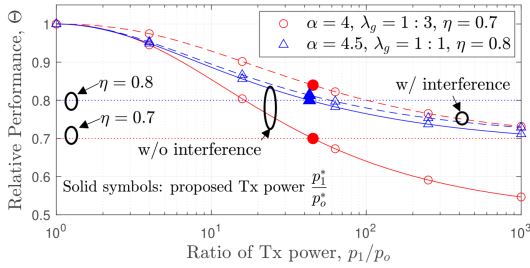


Fig. 2. Impact of interference on design performance.

racy of the proposed method. The values of S and S_g , determined by Lemma 1, align perfectly with the simulation results. As p_1 increases, S decreases. The optimal values p_1^* and p_2^* , easily obtained from the solution of simple $f_k(x) = 0$ in Theorem 1 or the closed form in Corollary 1, are marked with solid symbols on the graph and meet the minimum requirement ηS .

Fig. 2 examines the performance deviation of the proposed method, which applies the worst-case performance loss requirement from an interference-free case, compared to actual interference scenarios. As expected, the method sets the performance loss requirement conservatively under interference, ensuring robustness for dynamic environments. In scenarios with reduced interference impact (e.g., $\alpha = 4.5$) and more uniform node distributions (e.g., $\lambda_g = 1 : 1$), where $\mathbb{E}[P^{2/t}]$ and $\mathbb{E}[P]^{2/t}$ differ less, the method demonstrates both robustness and high accuracy. Thus, the proposed approach is highly effective in scenarios requiring robust performance loss guarantees and low complexity.

V. Conclusions

This paper proposed a method to jointly optimize node group transmit power, using a bisection search

on a simple function for general path loss exponents (PLEs) and a closed-form expression for a PLE of four, with the aim of enhancing a specific group's performance while limiting overall loss. The method imposed stricter conditions under interference but delivered robust performance across diverse scenarios, making it suitable for applications requiring robustness and low complexity. Future study will explore the impacts of more practical channel and node distribution models.

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